A new self-tuning fuzzy controller for vibration of a flexible structure subjected to multi-frequency excitations

Jie Fu¹, Peidong Li¹, Seung-Bok Choi², Guanyao Liao¹, Yuan Wang¹ and Miao Yu¹

Abstract
This work proposes a new self-tuning fuzzy controller for vibration control of a flexible structure subjected to both single-frequency and multi-frequency excitations. A piezoelectric stack actuator is used to generate control force to attenuate vibrations in the resonant and non-resonant frequency ranges. As a first step, modal frequencies and modal shapes of the flexible beam structure are obtained via the finite element method, and these are verified through comparison with the measured values. Second, in order to attenuate multi-frequency vibration of the beam, a new fuzzy controller with self-tuning factors is proposed. This controller is independent of the structure model and very adaptive to variable excitations. Then, vibration control performances of the self-tuning fuzzy controller are experimentally investigated under two different excitation conditions: single-frequency and multi-frequency excitations. It is shown that vibration control capability of the proposed controller outperforms conventional fuzzy controller under two different excitation conditions. In addition, it is demonstrated by experimentally implementing the proposed self-tuning fuzzy controller that high performances of multi-frequency vibration control are successfully achieved in which both amplitude and frequency of sinusoidal excitations are varied.

Keywords
Vibration control, multi-frequency excitation, self-tuning fuzzy controller, flexible structure, piezoelectric stack actuator, modal analysis

Introduction
Flexible structures in engineering fields are widely used: civil engineering (long-span bridge),¹ aerospace engineering (cabin of a space station or solar panels)² and mechanical engineering (machine structure).³,⁴ Despite several advantages of the flexible structures such as light weight, unwanted vibrations are inevitably occurred and this may cause instability of the system resulting in degraded performances and catastrophic failure. In order to suppress vibrations and improve safety, numerous types of passive mounts⁵ such as rubber mount and hydraulic mount have been developed. The rubber mount with low damping shows efficient vibration isolation performance in high-frequency and non-resonant excitations. However, it cannot provide a favorable performance in low-frequency and resonant excitations. The hydraulic mount with large damping is effectively utilized to isolate low-frequency vibrations, but not effective in high-frequency band.⁶ To overcome the disadvantages of passive mounts, active mounts...
utilizing smart materials such as shape memory alloys,\textsuperscript{7} magnetostrictive materials\textsuperscript{8} and piezoelectric materials\textsuperscript{9–14} have been actively studied in recent years. Among these active mounts, the piezoelectric actuator is featured by small stroke, fast response time, high force generation and wide working frequency bandwidth.\textsuperscript{15} Using these salient features, an effective vibration control performance of various flexible structure systems subjected to small-magnitude and high-frequency bandwidth excitations can be accomplished. In general, the performance of the dynamic responses of flexible structures highly depends on an appropriate control strategy. Choi et al. designed a linear quadratic Gaussian (LQG) controller to suppress the vibration of a flexible structural system with inertial type piezoelectric mount. They experimentally verified good controller performance by evaluating acceleration response of the structural system in frequency and time domains.\textsuperscript{6} Hanagud et al.\textsuperscript{16} developed an acceleration feedback controller for the piezoelectric stack actuator (PSA) to reduce the amplitude of aircraft’s tail buffet in a selected frequency range (8–80 Hz). Nitsche and Gaul used the PSA to control the normal force in the friction interface of a rotational joint connecting two linear elastic beams. They formulated a damping controller to dissipate the system energy additionally based on Lyapunov function.\textsuperscript{17} Choi et al. developed a hybrid mount consisting of elastic rubber and PSA for the effective vibration control of a flexible beam structure. In this work, a sliding mode controller was designed and implemented to attenuate the vibration of the structure subjected to high-frequency and small-magnitude excitations.\textsuperscript{18} It is noted here that in order to successfully achieve vibration control performance, accurate system models are absolutely required for the above-mentioned control methods. However, in practice, it is difficult to formulate an accurate mathematical or analytical model of the flexible structure due to several uncertainties such as parameter variation. Therefore, the fuzzy controller (FC) which is independent of complicated mathematical model of the structure system is frequently adopted.

One of salient features of the FC is that an expert knowledge is incorporated into the controller design process using some linguistic rules and this leads to a nonlinear controller which can describe any complex relationships between input and output variables. In addition, the FC is very robust against uncertainties.\textsuperscript{19–21} and hence, it has been extensively researched in various fields of engineering.\textsuperscript{22–25} Xu et al.\textsuperscript{26} designed a FC to control vibrations of the solar panel using the PSA and showed that the FC could effectively suppress the vibrations in a short time with appropriate choice of input and output scaling factors. However, for the sinusoidal excitations in which both the amplitude and frequency of the excitations are varied, conventional FC with the fixed quantification factors, scaling factors, rules and membership functions could not guarantee stable control performance. Therefore, the self-tuning FC is required to deal with this kind of excitation condition.\textsuperscript{27–31} It is well known that the gain-tuning FC provides much better control responses in real-time environment than the FC associated with the tuning of rule base and membership functions.\textsuperscript{27–31} Gao et al.\textsuperscript{31} proposed a self-tuning FC for a class of industrial temperature control system and showed significant improvement in maintaining stability over a wide range of fixed temperature. The self-tuning factor used in Gao et al.\textsuperscript{31} was adjusted by another fuzzy rule which could lead large computation costs, and hence, it was not suitable for real-time vibration control of flexible structures with multi-frequency excitations. Other research works on the gain-tuning FC showed only simulation results by focusing on the control strategy.\textsuperscript{11,29,30} As surveyed, the study on the gain-tuning FC associated with experiment verification is considerably rare. Moreover, it is still challenge to experimentally implement the gain-tuning FC for vibration control of flexible structures subjected to variable excitations in terms of amplitude and frequency.

Consequently, the main technical originality of this work is to propose a new FC with self-tuning factors and experimentally implement it to verify excellent vibration control performance of a flexible structure subjected to multi-frequency excitations. The control force associated with the new FC controller is generated by adopting the PSA and the flexible beam structure is subjected to both single-frequency and multi-frequency excitations. In section “Modal analysis,” the finite element method is utilized to obtain modal frequencies and mode shapes of the beam structure, and these are validated via experimental identification. In section “Design of FC with self-tuning factors,” considering a strong nonlinearity of the flexible beam structure, a self-tuning FC is designed considering self-tuning laws of the quantification factors and scaling factor obtained from the linear fitting. And in section “Experimental results and discussions,” experimental realizations of the proposed controller are undertaken and vibration control performances are presented in time and frequency domains. In addition, a comparative work between the proposed controller and conventional FC is performed under same experimental conditions. It is shown from experimental results that vibrations at the resonant and non-resonant frequency ranges are well controlled despite multi-frequency excitations in which both amplitude and frequency are varied during control action.

Modal analysis

It is necessary to obtain dominant resonance frequencies of the flexible beam structure before designing the controller. Therefore, both modal analysis based on finite element analysis and experimental identification are carried out to obtain modal frequencies and mode shapes of the beam.

Figure 1 shows the schematic configuration of the flexible beam structure incorporated with the PSA. The
The dimension of the beam (304 stainless steel material) is 600 mm (length) × 50 mm (width) × 2 mm (thickness), and the constraints used for the end beam is fixed support. The PSA is mounted just below the center of the beam denoted as position 2. And positions 1 and 3 are placed at 150 and 450 mm from the left end of the beam, respectively. The finite element analysis is performed using ANSYS software to identify modal frequencies and shapes. The results are shown in Figure 2. It is identified from the results that the maximum vibration amplitude of the beam is occurred at positions 1 and 3, and the first two-order modal frequencies are identified by 71 and 90 Hz, respectively.

The modal characteristics of the proposed flexible structure are also investigated via experiment to validate the results obtained from the finite element analysis. Figure 3 shows an experimental setup for modal test. The dSPACE/AutoBox (Model ds1005; dSPACE, Germany) generates a sine sweep signal with the frequency range of 20–300 Hz, which is amplified by the power amplifier (Model JF2; Chongqing University, China) to drive the exciter (Model JZ1; Far East Vibration, China). The response of the acceleration at position 1 is measured using an accelerometer (Model 333B52; Piezotronics, USA). It is remarked that in order to experimentally obtain the modal frequencies or resonant frequencies of the flexible structure, the accelerometer cannot be placed on the vibration nodes. From the results of ANSYS modal analysis, it is known that position 2 in Figure 1 is the vibration node. So the accelerometer should be placed at position 1 or 3. However, in this work, the accelerometer is mounted at position 3 since PSA is put on the node rather than position 1 or 3. If the PSA is mounted on position 1 or 3, the mounting point will be a new node instead of the position with the maximum amplitude. Another reason is that since the vibration magnitude at position 2 is very small, the PSA can be protected from the failure which may be caused from the high tensile stress. The basic performances of the PSA (Model 40VS12; Core Tomorrow, China) used in this work are experimentally investigated and shown in Figure 4. It can be seen clearly that the output displacement and force increase with the increase of the input voltage, and the frequency response of the output force and displacement are almost invariable within the frequency range of 100 Hz. Now, the frequency response function of the flexible beam structure is obtained using a sine sweep method and presented in Figure 5. As mentioned earlier, this response is measured at position 1 using an accelerometer. It is identified from this result that the dominant modes are the first mode (72 Hz) and second mode (91 Hz), which agree well to the analysis results. Thus, the first two modes will be considered as dominant modes of the flexible structure for vibration control.

Design of FC with self-tuning factors

The main idea of the FC is the use of linguistic instructions based on human skills as a basis for control. By transferring human skills into linguistic IF-THEN rules, the FC can be embedded into a closed-loop system, which is similar to conventional controllers. The synthesis of a conventional FC involves fuzzifier, fuzzy rules and fuzzy inference and defuzzifier, as shown in Figure 6. The input variables to the developed FC are selected as the error of acceleration $e$ and its rate of change $e_c$, while the output is chosen as the applied voltage to the PSA. The error signal is obtained by

$$e = a_r - a_m$$

where $a_m$ is the acceleration measured by the accelerometer, $a_r$ is the reference value, which is set as zero in this work, and $e$ denotes the error of acceleration.

Fuzzifier

The fuzzifier converts the input variables from numerical form into linguistic form. The structure of FC will be more complex if many linguistic variables are chosen.
to describe inputs or outputs. However, the lesser ones will cause worse performance of the system. In this work, considering the controller’s complexity and accurate performance of the system, the input and output linguistic variables are assigned by seven fuzzy subsets. These are denoted by negative big (NB), negative medium (NM), negative small (NS), zero (ZO), positive small (PS), positive medium (PM) and positive big (PB). The discourse domain of the inputs is set as \([-6, 6]\). Hence, the input variables need to be scaled in such a manner that their minima are \(-6\) and maxima are 6

\[
\begin{align*}
    k_e &= \frac{e_{\text{max}}}{E_{\text{max}}} \\
    k_{ec} &= \frac{e_{c\text{max}}}{E_{c\text{max}}}
\end{align*}
\]  

(2)

where \(k_e\) and \(k_{ec}\) represent the quantification factor of the error and the error’s change, respectively. \(e_{\text{max}}\) and \(e_{c\text{max}}\) are the maxima of the input variables, and \(E_{\text{max}}\) and \(E_{c\text{max}}\) are the maxima of the discourse domain of the inputs. In conventional FC, the quantification factors are normally set based on specific signals and thus constant.

Fuzzy rules and fuzzy inference

Fuzzy rules play an important role in a fuzzy control system. In this work, for the active vibration control system, \(E\) and \(EC\) are selected as PB, and \(U\) (the output of the FC) is selected as NB to prevent the change of \(E\). If \(E\) is PB and \(EC\) is negative, \(U\) should be NS or ZO to keep the system stable. So the \(i\)th rule can be written as follows

\[
\text{If } E = A_i \text{ and } EC = B_i, \text{ then } U = C_i, \quad i = 1, 2, \ldots, 49
\]  

(3)

where \(A_i, B_i, C_i\) are the linguistic values of the fuzzy variables. Considering the number of fuzzy subsets given for each input and output in the fuzzifier section, it can be concluded that the number of fuzzy rules is 49, and the rules are listed in Table 1.

The fuzzy inference of the controller is based on Mamdani’s method, which is associated with the max–min composition. The membership functions of the input and output are illustrated in Figures 7 and 8. It is noted that an “S”-shaped membership function is selected for NB and PB to enhance the smoothness of
the control processing and guarantee the stability of

the controlled system. And different triangle-shaped membership functions are used for the rest of linguistic variables to improve the sensitivity of the controlled system, especially under small-magnitude excitations.

Table 1. Fuzzy rules of the designed FC.

<table>
<thead>
<tr>
<th>U</th>
<th>NB</th>
<th>NM</th>
<th>NS</th>
<th>ZO</th>
<th>PS</th>
<th>PM</th>
<th>PB</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>ZO</td>
<td>PM</td>
<td>PM</td>
<td>PS</td>
<td>PM</td>
<td>PS</td>
<td>PS</td>
</tr>
<tr>
<td>E</td>
<td>PS</td>
<td>PM</td>
<td>PM</td>
<td>ZO</td>
<td>NS</td>
<td>NS</td>
<td>NS</td>
</tr>
<tr>
<td>E</td>
<td>NS</td>
<td>PB</td>
<td>PB</td>
<td>PM</td>
<td>PS</td>
<td>PS</td>
<td>ZO</td>
</tr>
<tr>
<td>E</td>
<td>PM</td>
<td>PS</td>
<td>PS</td>
<td>NS</td>
<td>NM</td>
<td>NM</td>
<td>NM</td>
</tr>
</tbody>
</table>

NB: negative big; NM: negative medium; NS: negative small; ZO: zero; PS: positive small; PM: positive medium; PB: positive big.

Figure 6. Structure of a conventional FC.

Defuzzifier

Through a process referred to as defuzzifier, the results of the fuzzy inference are transformed back to numerical form to be implemented in practical control. The center-of-gravity method is adopted as the defuzzification technique to compute the output voltage $U$, which is widely utilized in the Mamdani inference method-based fuzzy control system.\(^{32}\) Since the discourse domain of the output $U$ is set as $[-7, 7]$, it must be scaled prior to its control action on the PSA

$$u = k_u \text{sgn}(U)$$

where

$$\text{sgn}(U) = \begin{cases} 1 & U \geq 0 \\ 0 & U < 0 \end{cases}$$

where $k_u$ represents the output scaling factor and $u$ is the control voltage to be applied to the PSA. A sign function (sgn()) is used here to guarantee that the voltage $u$ is always non-negative to protect the PSA, which cannot tolerate negative driving voltages. In other words, this indicates a semi-active input voltage which always guarantees the stability of the proposed control system.

Factor adjustment mechanism

When the frequency and amplitude of the excitation signal are known, the calculated quantification factors and scaling factor according to equations (2) and (4) are constant. However, there is variation of excitation in many practical applications. In this case, the FC with constant factors may not be able to achieve satisfying control effect. To improve control performance, the factor adjustment mechanisms needs to be designed on the basis of the excitation estimate result. A new structure of the FC with self-tuning factors is shown in Figure 9.

The peak observer first computes and stores local maxima or minima $P$ of the input signal $x(t)$, which can be obtained by the following equation

$$P = x(t - T), \quad |x(t) - x(t - T)|$$

$$|x(t - T) - x(t - 2T)| = 0$$

where $T$ is the sampling period of the controlled system. Then, the absolute maximum $A(t)$ in $n$ cycles can be expressed by

$$A(t) = \max([|P_1|, |P_2|, \ldots, |P_n|])$$

where $n$ is determined by the experimental test for variable excitation which is 15 in this work. The observer in equation (6) is sensitive to the noise, and hence, the input error is preprocessed by a band-pass filter. In Figure 9, the peak observer identifies the peak value of the error $A_e(t)$ and the one of the error’s change rate $A_{ee}(t)$ on-line. And the quantification factor and scaling factor adjustment mechanism (adaptation law) are determined as follows.
\[
\begin{align*}
    k_e(t) &= \frac{F_{\text{max}}}{A_e(t)} \\
    k_{ec}(t) &= \frac{E_{\text{max}}}{A_{ec}(t)} \\
    k_w(t) &= \gamma A_e(t)
\end{align*}
\]

where \( A_e(t) \), \( A_{ec}(t) \) are obtained by the peak observer, and \( \gamma \) is identified to be 0.016 through the linear fitting based on the measured values, as shown in Figure 10.

The effectiveness of this method will be verified in the following section.

**Experimental results and discussions**

In order to evaluate vibration control performance of the proposed self-tuning FC, an experimental setup is established, as shown in Figure 11. The desired waveform is generated from the signal generator (Model 33120A; Agilent, USA) and amplified by the power amplifier to drive the exciter. The acceleration of the beam at position 3 is measured by an accelerometer. After being conditioned and filtered, the acceleration signal is sent to the designed self-tuning FC, which is realized in the MATLAB/Simulink environment followed by the downloading to the dSPACE/AutoBox. The piezoelectric amplifier (Model XE501A; Core Tomorrow) is used to supply high voltage to the PSA to achieve enough actuating force. By considering the limitation of computing speed of AutoBox and vibration modes to be controlled, the sampling rate in the experimental implementation is chosen by 600 Hz.

**Single-frequency excitation**

The excitation signals used in this work are denoted by acceleration and frequency as follows: acceleration amplitude—0.5, 1.0 and 1.5 m/s\(^2\) and frequency change—60, 70, 80 and 95 Hz. The used frequency covers resonant frequencies and non-resonant frequencies. The quantification and scaling factors in the conventional FC are regulated with the amplitude of 0.5 m/s\(^2\) and keep constant. Table 2 lists the factors of both conventional FC and self-tuning FC under different excitations. Vibration control performances of conventional FC and the proposed controller are measured at different excitations and presented in Figures 12–15. It is clearly observed that control performance of the proposed self-tuning FC is superior to that of conventional FC as expected. The root-mean-square (RMS) values and percentage decrement of the accelerations at position 3 are calculated based on the result of the
Figure 12. Experimental results under 0.5 m/s\(^2\) and 80 Hz excitation: (a) measured acceleration response and (b) control voltage.

Figure 13. Experimental results under 1.0 m/s\(^2\) and 80 Hz excitation: (a) measured acceleration response and (b) control voltage.

Figure 14. Experimental results under 1.5 m/s\(^2\) and 70 Hz excitation: (a) measured acceleration response and (b) control voltage.

Figure 15. Measured results under 1.5 m/s\(^2\) and 95 Hz excitation: (a) measured acceleration response and (b) control voltage.
uncontrolled case and given in Table 3. From these results, the followings can be identified: (1) conventional FC with constant factors could only achieve satisfying control effect when the excitation signal is constant or a small variation. For instance, when the acceleration amplitude is two times bigger than 0.5 m/s², the percentage decrement is just over 10%. (2) The self-tuning FC results in greater percentage decrement of acceleration RMS values showing around 40% in both resonant and non-resonant frequencies. To sum up, it clearly indicates that the factor adjustment mechanism proposed in this work performs well and the self-tuning FC could effectively attenuate the variable single-frequency vibrations.

**Multi-frequency excitation**

To further demonstrate the effectiveness of the proposed FC with self-tuning factors, the response accelerations at position 3 are measured under multi-frequency excitations. The excitations consist of four different frequency compositions: 70 + 80 Hz, 70 + 95 Hz, 80 + 95 Hz and 70 + 80 + 95 Hz, and the peak value in time domain of each excitation

---

**Table 2.** Quantification and scaling factors of the conventional and self-tuning FCs in each case under single-frequency excitations.

<table>
<thead>
<tr>
<th>Acceleration amplitude (m/s²)</th>
<th>Excitation frequency (Hz)</th>
<th>Conventional FC</th>
<th>Self-tuning FC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$k_a$</td>
<td>$k_{ac}$</td>
</tr>
<tr>
<td>0.5</td>
<td>60</td>
<td>12</td>
<td>0.0318</td>
</tr>
<tr>
<td>1.0</td>
<td>80</td>
<td>6</td>
<td>0.0136</td>
</tr>
<tr>
<td>1.0</td>
<td>95</td>
<td>4</td>
<td>0.0099</td>
</tr>
<tr>
<td>1.5</td>
<td>70</td>
<td>4</td>
<td>0.0099</td>
</tr>
<tr>
<td>1.5</td>
<td>95</td>
<td>4</td>
<td>0.0067</td>
</tr>
</tbody>
</table>

FC: fuzzy controller.

**Table 3.** Comparison of the accelerations at position 3 under different excitations and controllers.

<table>
<thead>
<tr>
<th>Acceleration amplitude (m/s²)</th>
<th>Excitation frequency (Hz)</th>
<th>Uncontrolled RMS (m/s²)</th>
<th>Conventional FC RMS (m/s²) Decrease (%)</th>
<th>Self-tuning FC RMS (m/s²) Decrease (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>60</td>
<td>0.3536</td>
<td>0.2196</td>
<td>38.04</td>
</tr>
<tr>
<td>0.5</td>
<td>80</td>
<td>0.2525</td>
<td>0.2525</td>
<td>28.59</td>
</tr>
<tr>
<td>1.0</td>
<td>70</td>
<td>0.7071</td>
<td>0.5657</td>
<td>20.00</td>
</tr>
<tr>
<td>1.0</td>
<td>80</td>
<td>0.6437</td>
<td>0.6437</td>
<td>20.00</td>
</tr>
<tr>
<td>1.0</td>
<td>95</td>
<td>0.6069</td>
<td>0.6069</td>
<td>14.17</td>
</tr>
<tr>
<td>1.5</td>
<td>70</td>
<td>1.0607</td>
<td>0.9210</td>
<td>13.17</td>
</tr>
<tr>
<td>1.5</td>
<td>95</td>
<td>0.9137</td>
<td>0.9137</td>
<td>13.17</td>
</tr>
</tbody>
</table>

FC: fuzzy controller; RMS: root mean square.

**Table 4.** Quantification and scaling factors of the conventional and self-tuning FCs in each case under multi-frequency excitations.

<table>
<thead>
<tr>
<th>Frequency composition (Hz)</th>
<th>Acceleration peak value (m/s²)</th>
<th>Conventional FC</th>
<th>Self-tuning FC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$k_a$</td>
<td>$k_{ac}$</td>
<td>$k_u$</td>
</tr>
<tr>
<td>70 + 80</td>
<td>0.5</td>
<td>12</td>
<td>0.0269</td>
</tr>
<tr>
<td>70 + 80</td>
<td>1.0</td>
<td>6</td>
<td>0.0138</td>
</tr>
<tr>
<td>70 + 80</td>
<td>1.5</td>
<td>4</td>
<td>0.0099</td>
</tr>
<tr>
<td>70 + 95</td>
<td>0.5</td>
<td>12</td>
<td>0.0231</td>
</tr>
<tr>
<td>70 + 95</td>
<td>1.0</td>
<td>6</td>
<td>0.0118</td>
</tr>
<tr>
<td>70 + 95</td>
<td>1.5</td>
<td>4</td>
<td>0.008</td>
</tr>
<tr>
<td>80 + 95</td>
<td>0.5</td>
<td>12</td>
<td>0.0224</td>
</tr>
<tr>
<td>80 + 95</td>
<td>1.0</td>
<td>6</td>
<td>0.0114</td>
</tr>
<tr>
<td>80 + 95</td>
<td>1.5</td>
<td>4</td>
<td>0.0075</td>
</tr>
<tr>
<td>70 + 80 + 95</td>
<td>0.5</td>
<td>12</td>
<td>0.024</td>
</tr>
<tr>
<td>70 + 80 + 95</td>
<td>1.0</td>
<td>6</td>
<td>0.0121</td>
</tr>
<tr>
<td>70 + 80 + 95</td>
<td>1.5</td>
<td>4</td>
<td>0.0081</td>
</tr>
</tbody>
</table>

FC: fuzzy controller.
composition is set as 0.5, 1.0 and 1.5 m/s², respectively. Each frequency component of multi-frequency excitations corresponds to each different fuzzy variable of conventional FCs. It is noted that constant quantification factors and scaling factors of each fuzzy variable are optimized for small-amplitude excitation (0.5 m/s²). Table 4 lists the factors of both conventional FC and self-tuning one under different excitations. Experimental results are presented in both time and frequency domains, as shown in Figures 16–19. Table 5 calculates the RMS values and percentage decrement of the accelerations in each case, while Table 6 presents the peak values of acceleration frequency spectrums under different excitations and controllers. Now, from these results, the followings are identified: (1) For the conventional FC with constant factors, the percentage decrement of RMS acceleration is about 20%, and the control effect is limited. Especially, the peak value of the acceleration is decreased only 2.27% at worst case, while the one using the FC with self-tuning is decreased...
up to 46.37%. (2) For the FC with self-tuning factors, the peak observer estimates the excitation changes online and the factors are adjusted according to equation (7). The RMS value of the acceleration decreased by 44.32%, while the one using the conventional FC reduced by 27.14%. (3) In general, the vibration control effect is more eminent in the resonant frequencies than the non-resonant frequencies as expected.

**Conclusion**

In this work, a new self-tuning FC was designed and experimentally realized for vibration control of a flexible structure in which the PSA was used as an actuator. As a first step, the first two modal frequencies of the flexible structure were calculated from the finite element analysis and also measured by sine sweeping method in frequency domain. Subsequently, a new FC with self-tuning factors was formulated for effective vibration control under multi-frequency excitations and experimentally realized using dSpace and other control devices such as filter, voltage amplifier and signal generator. It has been demonstrated that the self-tuning FC can attenuate unwanted vibrations of the flexible beam structure under both single-frequency and multi-frequency excitations. More specifically, about 40% decrement of the RMS acceleration has been achieved for both excitations using the proposed controller, while it is less than 20% using the conventional FC with constant tuning factors. It is noted that

![Experimental results under 1.0 m/s² and 70 Hz + 80 Hz excitation](image1)

![Experimental results under 1.5 m/s² and 70 Hz + 95 Hz excitation](image2)

![Experimental results under 1.0 m/s² and 80 Hz + 95 Hz excitation](image3)
the most salient benefit of the proposed controller is a potential capability of vibration control of flexible structures subjected to multi-frequency excitations in which both amplitude and frequency are varied during control action. This is possible due to the use of the adjustment mechanism in which tuning factors (adaptation laws) are determined in real time based on the excitation estimation result achieved from the observer.

The results presented in this work are quite self-explanatory justifying that the proposed self-tuning FC can be effectively applied to vibration control of various flexible dynamic systems subjected to multi-frequency excitations in which both amplitude and frequency are changed during control action. It is finally remarked that the control performance of the proposed controller heavily depends on the excitation estimation result from the observer and the output scaling factor of $k_u$, which finally determines the magnitude of the input voltage to the PSA. The computation time, of course, depends on the characteristics of the excitations. Therefore, in order to achieve vibration controllability in high-frequency excitations (say above 300 Hz in this work), an optimization of the observer design and scaling adjustment mechanism should be undertaken considering the high-frequency excitations to reduce the computation time.

Declaration of conflicting interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This research was supported by Chongqing Research Program of Basic Research and Frontier Technology (grant no. cstc2015jcyBX0069), the Natural Science Foundation of Chongqing (grant no. cstc2015jcjA0848) and the Fundamental Research Funds for the Central Universities (grant no. CDJZR13120090). The authors are grateful for the helpful comments from the reviewers and editor to improve this article.

References


