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NARX neural network modeling and robustness analysis of magnetorheological elastomer isolator

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Abstract
Due to the controllability of the stiffness and damping under the applied magnetic field, magnetorheological elastomer isolator has been proved effective in the field of vibration control. For the realization of vibration control application, an accurate MRE isolator model is a non-trivial task. However, the existing parametric modeling methods are required to identify too many parameters, which are difficult to implement. Moreover, the corresponding inverse dynamic model of the isolator cannot even be obtained by the identified model inversion. Therefore, this paper proposes a nonparametric neural network approach to approximate the dynamic behaviors of magnetorheological elastomer isolator with the characteristics of nonlinearity and hysteresis. Firstly, the dynamic characteristics of the isolator in shear-compression mixed mode are experimentally tested under different loading conditions. Secondly, based on the experimental data, a NARX neural network with three-layer structure is developed to approximate the functional relationship between inputs (displacement, velocity and current) and output (force) of magnetorheological elastomer isolator. Thirdly, the effectiveness of the network model is validated by comparing the predicted force and experimental force. Finally, considering the common occurrence of inputs with noise disturbance in real application, the robustness of the network is also verified for displacement and current inputs with noise disturbance, respectively. The results of the network generalization for experimental data show that the proposed NARX network is more robust and optimal than BP network.

Keywords: magnetorheological elastomer isolator, modeling, hysteresis, robustness

(Some figures may appear in colour only in the online journal)

1. Introduction
Magnetorheological elastomer (MRE) is a new kind of smart material, composed of elastomeric rubber medium and micron-sized ferromagnetic particles [1, 2]. Different from the rubber materials, the mechanical properties (stiffness and damping) of MRE are controllable under the applied external magnetic field [3–7]. It is found that there is a greater magnetorheological effect for MRE at the smaller strain amplitude [2], so the MRE material has a greater potential for the application of micro-vibration control [8, 9].

The model of MRE isolator in practical application of control is significantly important, but it is hindered to implement by their inherently hysteretic and highly nonlinear dynamics [10]. In addition, because of the limits of structure design method and repetitive test condition, existing models are mainly focused on material research [11–14]. There are only a few papers involving the model of MRE isolator [15–22], and most of them are parametric models. Yu et al [15] employed the Kelvin model to characterize the performance of MRE isolator under different magnetic fields. However, the model cannot effectively describe the hysteresis of the
isolator. Du et al [16] proposed a Bouc–Wen model to portray the MRE isolator’s dynamic behaviors and explained the influence of different model parameters on the hysteresis shape. Gordaninejad et al [17] proposed an improved Bouc–Wen model for the MRE-based isolator by adding a standard three-element solid model in parallel. Li et al [18] proposed a strain-stiffening model for MRE-based isolator, which consisted of a standard three-element solid model and a modified Maxwell model. In addition, hyperbolic sine function model [19], improved Dahl model [20] and LuGre friction model [21] are also used for the purpose. Though the models mentioned can describe the hysteretic behavior of MRE isolator, there are too many parameters to be identified. Moreover, the corresponding inverse dynamic models of MRE isolator are difficult to obtain due to their highly nonlinearity. Therefore, parametric models are not easy to incorporate in existing control frameworks.

Neural network (NN) is an effective non-parametric modeling method to approximate any nonlinear mapping relationship [22], which is not able to be solved by a mathematical model. The advantages of NN are high accuracy, low complexity and easy identification. To date, relevant researches on NN-based model of MRE isolator are few. Only Yu et al [23] proposed a feedforward NN with ant colony algorithm to modeling the laminated MRE base isolator. However, the feedforward network is a static network, and has lower accuracy and poorer capacity of resisting disturbance. So, it is not suitable for on-line control system, especially for the system with noise disturbance. Actually, when MRE isolator is modelled, noise disturbance existing in inputs and outputs is unavoidable to be considered. Because signal measurement and transmission are noisy processes, the existing noise will lead to inaccurate model. Though the noise can be dealt with by employing noise filter, it is invalid in the case of same frequency between input signals and noise. Therefore, it is of practical importance to study the noise effects on the modelling accuracy. However, few results related to this problem have been published recently. Recurrent NN provides an effective solution for the problem. It is a dynamic network where connections between units form a directed cycle. This creates an internal state of the network which allows it to exhibit dynamic temporal behavior. Unlike feedforward NN, recurrent NN can use their internal memory to process arbitrary sequences of inputs. So it has higher accuracy and stronger robustness, and is also beneficial for on-line control [24]. The nonlinear autoregressive with external input (NARX) network is a kind of recurrent NN with a limited feedback which comes only from the output neuron rather than from hidden states. It is found that NARX network typically converges faster and generalizes better than these other recurrent NNs [25]. Thus, the NARX network is employed for the model of MRE isolator in this paper.

In this paper, the dynamic characterization test system of the MRE mixed mode isolator is established, and the output forces of the isolator are obtained under different loading conditions (variable amplitudes and frequencies, and variable applied currents) in section 2. In section 3, based on the experimental data and trial and error method, the configuration of the network is determined to guarantee a smaller training error, which has a three-layer structure of 11 input neurons (displacement, velocity, the command current and their delay values, previous forces), 7 hidden neurons and 1 output neuron (force). Then, the model of the MRE isolator is identified, and the effectiveness of the network is also validated by comparing the predicted force and experimental force. The results show that the approximation capability of proposed NARX network outperforms BP network. In section 4, the robustness of the NARX network is verified for inputs (displacement and current) with noise disturbance. In section 5, the conclusions accentuate that the proposed NARX network is of higher accuracy, stronger robustness and more benefit for on-line control of MRE isolator.

2. Dynamic characteristic experiment of MRE isolator

2.1. Experimental setup

The structure of MRE isolator in shear-compression mixed mode in the paper is shown in figure 1, which consists of seven parts. There are two pieces of silicone rubber MREs in which the mass fraction of carbonyl iron particles, silicon rubber and silicone oil is 70%, 15% and 15%, respectively. One piece operates on shear mode with the size of 40 × 4 × 4 mm, and the other works on compression mode with the diameter of 20 mm and a thickness of 4 mm. When the vertical vibration occurs, the movable plate moves up and down, which causes the former MRE sheared and the latter compressed or tensioned. The advantage of shear-compression mixed mode MRE isolator is that the static stress of the system caused by the isolation mass can be effectively reduced by the compression mode, and the wide regulating range of stiffness is produced by the shear mode. The limit device connected with iron core is used to provide a groove for coil to cut off the magnetic circuit. The iron core, MREs, movable plate and mounting shell form a closed magnetic circuit, which is the red close loop in figure 1.

According to the simulated results, the total magnetic flux density in MRE isolator is 581.9 mT. More detail illustration on the isolator design and magnetic circuit analysis can be seen in Ref. [15].
In order to evaluate and characterize the performance of the MRE isolator, a series of experimental tests are conducted under various loading conditions. The experimental equipment is shown in figure 2. The electromagnetic vibration table (model MPA406/M232A; ETS Solutions (Beijing) Led, China) is used to supply an excitation signal for the isolator. The MRE isolator is mounted on the vibration table and moves along with the vibration table motion. A displacement sensor (model LK-H025; KEYENCE, Japan) is fixed on the vibration table to measure the displacement of the base. A force sensor (BISE5110, No.0748) connected to the top of the isolator is fixed to the three-jaw chuck to measure the force signal generated by the isolator. The magnetic coil is energized by adjusting the DC power which provides the DC current from 0 to 1.5 A at the interval of 0.5 A. The excitation signal and the response signal are collected by a data acquisition instrument (model MDR-05, Beijing Aero-standard New Technology Company) and transferred to the computer for processing and analyzing.

Figure 2. Experimental equipment: (a) sketch (b) picture.

![Experimental equipment](image)

Figure 3. Force responses of the MRE isolator with sinusoidal loading (0.045 mm-40 Hz).

![Force responses](image)

By applying various harmonic inputs with different coil currents, the response forces of the isolator can be recorded. To make the identified model fully represent the MRE
isolation system, the modeling data should cover all possible ranges of input variation. So, three amplitudes (0.045 mm, 0.09 mm and 0.13 mm, corresponding to 2.3%, 4.5% and 6.5% shear strain) are selected for the tests under various loading frequencies (40, 60, 80 and 100 Hz). For each loading case, four currents (0.0, 0.5, 1.0 and 1.5 A) are applied to examine performance of the isolator under different magnetic fields. In each test, the sampling frequency for the data acquisition is set to 2048 Hz for capturing all test results. The velocity response is calculated by the differential of the measured displacement.

2.2. Experimental results and analysis

Figure 3 shows the force responses of the MRE isolator under a sinusoidal loading of 0.045 mm amplitude and 40 Hz frequency with various applied currents (0 A, 0.5 A, 1 A and 1.5 A). As clearly indicated from the figure, the measured force has a significant increase with increasing of applied currents from 0 A to 1.5 A.

The force-displacement and force-velocity loops are shown in figure 4, in which the isolator is driven with 0.045 mm amplitude, 60 Hz frequency and four current levels (0, 0.5, 1 and 1.5 A). Figure 5 displays the hysteresis loops of the isolator at the constant amplitude of 0.09 mm and current of 1 A under various frequencies (40, 60, 80 and 100 Hz). Figure 6 illustrates the force-displacement and force-velocity relationships of the isolator with 1.5 A applied current and 80 Hz frequency under different loading amplitudes (0.045, 0.09 and 0.13 mm).

The effective stiffness values (calculation method referred to the appendix) and changes under different loading conditions (applied current, excitation frequency and amplitude) are listed in table 1 and figure 7, respectively. It can be known from the table and figure that there is a significant increase of the effective stiffness with the increasing of applied current for the isolator, and the value has a decrease with the increasing of loading frequency or amplitude. For a fixed frequency 40 Hz at the constant amplitude of 0.045 mm, the maximum increment of effective stiffness can reach 107.8% in table 1.

The equivalent damping values under different loading conditions are also given in table 2 and figure 8. For a given frequency 40 Hz at the 0.045 mm amplitude, with the current increasing from 0 A to 1.5 A, the maximum relative increase of equivalent damping is 175.6% in table 2. It can be seen that the variety of equivalent damping is similar to that of effective stiffness. That is, the equivalent damping is proportional to the applied current, and is inversely proportional to the excitation displacement amplitude and frequency.

According to the above analysis, the effective stiffness and equivalent damping are not only affected by applied current, but also related to the excitation displacement amplitude and frequency. Therefore, the nonlinear and hysteretic behaviors of the isolator are further illustrated by experimental results.

Based on the above experimental data, a recurrent NN-based forward model of the MRE isolator will be established in the following section.

3. NARX network modeling and validation for MRE isolator

The NARX network is employed to describe the functional relationship between output force and displacement, velocity and applied current in the paper.

3.1. NARX network

The NARX network with the structure of input layer, output layer and hidden layer is shown in figure 9, in which tapped delay lines (TDL) are used for inputs including exogenous variables and feedback variable from output neuron. In view
of the inputs and output variables of MRE isolator, the dynamic equation for the NARX network is given by

\[
\dot{F}_t = f(x(t), x(t-1), \ldots, x(t-n_x), \dot{x}(t), \dot{x}(t-1), \ldots, \dot{x}(t-n\dot{x}), I(t), I(t-1), \ldots, I(t-n_I), \hat{F}(t-1), \ldots, \hat{F}(t-n_F))
\]  

(1)

where \(x(t-m), \dot{x}(t-l), I(t-h)\) and \(\hat{F}(t-g)\) \((m = 0, 1, \ldots, n_x, l = 0, 1, \ldots, n\dot{x}, h = 0, 1, \ldots, n_I\) and \(g = 1, 2, \ldots, n_F\)) are the inputs of the network, and they are the displacement, velocity, current and output force at time \(t-m, t-l, t-h\) and \(t-g\), respectively. \(\{n_x, n\dot{x}, n_I, n_F\}\) is the set with four positive integers, which represents the maximum input delay. \(f(\cdot)\) is a continuous nonlinear function. \(\hat{F}(t)\) is the network’s estimated output force at time \(t\), which not only depends on the present input values, but also the previous inputs and output values.

The algorithm of real time recurrent learning (RTRL) \([26]\) is adopted to adjust the weights of NARX network based on gradient descent method and expanded back propagation algorithm. The detailed process is given as follows:

**Step 1.** Apply the training samples to network and initialize the parameters. The initial weights are produced randomly, while, the training parameters such as epochs, goal and the size of network are set according to the demand.

**Step 2.** Calculate the output. The output of \(j\)th node at time \(t\) is calculated by the following equation

\[
\hat{F}_j(t) = \varphi(v_j(t)), j \in B
\]  

(2)
where \( \varphi[\cdot] \) denotes the unit's activation function, and the net input \( v_j(t) \) at time \( t \) is

\[
v_j(t) = \sum_{i \in A \cup B} w_{ji}(t) u_i(t) - \theta_j
\]  

\( w_{ji}(t) \) is the weight of \( j \)th node with \( i \)th input, \( \theta_j \) is the threshold of \( j \)th neuron, and the total input \( u_i(t) \) is described as

\[
u_i(t) = \begin{cases} 
   x(t-i) & i \in A_1 \\
   \dot{x}(t-i) & i \in A_2 \\
   I(t-i) & i \in A_3 \\
   \dot{F}(t-i) & i \in B
\end{cases}
\]

where \( A \) and \( B \) are the sets of exogenous input and feedback input, respectively. The set of \( A \) contains three subset of \( A_1, A_2 \) and \( A_3 \) which correspond to the displacement, velocity and current.

**Step 3.** Calculate the increment of the weights. The overall network error at time \( t \) is

\[
J(t) = \frac{1}{2} \sum_{j \in B} (e_j(t))^2
\]

where the error \( e_j(t) \) of \( j \)th node at time \( t \) is described as

\[
e_j(t) = F_j(t) - \hat{F}_j(t)
\]

\( F_j(t) \) is the target value of \( j \)th node at time \( t \). The criterion in learning process is to minimize the total error. According to the gradient descent algorithm, each weight is updated using the increment

\[
\Delta w_{ji}(t) = -\eta \frac{\partial J(t)}{\partial w_{ji}} = \eta \sum_j e_j(t) p_{ji}^j(t)
\]

---

**Table 1.** Effective stiffness (kN/m) of the MRE isolator under different loading conditions.

<table>
<thead>
<tr>
<th>Current (A)</th>
<th>Frequency (Hz)</th>
<th>0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>Increase (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Amplitude A = 0.045 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>106.02</td>
<td>157.53</td>
<td>193.73</td>
<td>220.29</td>
<td>107.8</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>95.94</td>
<td>139.52</td>
<td>168.20</td>
<td>200.13</td>
<td>108.6</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>87.02</td>
<td>117.09</td>
<td>139.58</td>
<td>167.68</td>
<td>92.7</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>75.44</td>
<td>90.72</td>
<td>99.52</td>
<td>118.25</td>
<td>56.7</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Amplitude A = 0.09 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>102.88</td>
<td>121.39</td>
<td>148.95</td>
<td>168.66</td>
<td>63.9</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>90.77</td>
<td>107.37</td>
<td>131.76</td>
<td>146.35</td>
<td>61.2</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>80.60</td>
<td>93.41</td>
<td>107.84</td>
<td>124.07</td>
<td>53.9</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>66.28</td>
<td>72.73</td>
<td>84.91</td>
<td>93.33</td>
<td>40.8</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Amplitude A = 0.135 mm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40</td>
<td>91.30</td>
<td>108.10</td>
<td>127.30</td>
<td>142.78</td>
<td>56.4</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>81.66</td>
<td>99.62</td>
<td>110.74</td>
<td>123.78</td>
<td>51.6</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td>74.25</td>
<td>83.41</td>
<td>93.30</td>
<td>105.17</td>
<td>41.6</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>57.05</td>
<td>64.16</td>
<td>71.08</td>
<td>74.52</td>
<td>30.6</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 7.** The change of effective stiffness: (a) Effective stiffness versus current; (b) Effective stiffness versus frequency; (c) Effective stiffness versus displacement.
where \( \eta (0 < \eta \leq 1) \) represents learning rate of weight and the variables of \( \{ p_{ij}(t) \} \) is defined as

\[
p_{ij}(t) = \frac{\partial \hat{F}_l(t)}{\partial w_{ij}(t)}, \quad \forall k \in B, \; j \in B \text{ and } l \in A \bigcup B \tag{8}
\]
The force $p_{ij}^l(t)$ can be computed by the following equation

$$p_{ij}^l(t + 1) = \varphi^l(v_j(t)) \sum_{i \in B} w_{ji}(t)p_{ij}^l(t) + \sigma_{ij}u_i(t)$$

with the initial condition

$$p_{ij}^l(0) = 0$$
Step 4. Update the weights. The revised equation of the weights is given by

\[ w_{kl}(t + 1) = w_{kl}(t) + \Delta w_{kl}(t) \]  

Step 5. Repeat the process until the error calculated on the basis of updated weights reach the training goal.

The procedure of RTRL algorithm is described in figure 10.

3.2. Modeling of MRE isolator with NARX network

The MRE isolator’s model identification scheme with NARX network is shown in figure 11. The inputs of the network are displacement, velocity, applied current together with their delay values and previous force, while the output is present force. The experimental data used to model the isolator are divided into two parts. One part is used as the training set to build the model, and the training data is one-and-a-half-cycle samples in each excitation case (various amplitude and frequency) as shown in figure 12. The other part is the testing set of one-cycle samples used for generalizing the network.

The parameters (time delay number, hidden layer number and neuron number) of NARX network are required to determine before network training. The optimal neuron number \( m \) in hidden layer can be obtained by the following equation [27]

\[ m = \sqrt{n + p + \alpha} \]  

where \( n (n \geq 1) \) and \( p (p \geq 1) \) is the neuron number of input layer and output layer, respectively. \( \alpha \) is a regulating constant with the range of [1, 10]. So, based on the equation (13), the neuron number \( m \) in each hidden layers lies in the range of [3, 13].

In order to obtain the optimal time delay and hidden layer number of the NARX network, simulations are carried out under a smaller enough training error. It is assumed that the maximum time delay of the four input variables is the same. In view of the complexity of the network, the maximum time delay is no more than 3. Figure 13 shows the relationship between mean squared error (MSE) and neuron number under different hidden layers and time delays. From figure 13, it can be known:

1. The network with 2 time delay always has a smaller MSE than others for any neuron number.
2. When the neuron number is larger than 7, there is almost no difference of the MSE under the different hidden layers and 2 delays.

Considered computation complexity and training time, the simpler network structure should be selected when the training errors of network are the same. Therefore, the three-layer structure NARX network with 11 input neurons, 7 hidden neurons, 1 output neuron and 2 input delays is determined.

Based on the determined network, the training data shown in figure 12 is inputted into the NARX network. After 20 s of training, the results are obtained in figure 14. The comparison between predicted forces and experimental forces is shown in figure 14(a). It can be observed that the predicted...
Figure 15. Comparison between predicted values and real values (0.045 mm-60 Hz): (a) force-displacement relationships; (b) force-velocity relationships.

Figure 16. Comparison between predicted values and real values (0.09 mm-1 A): (a) force-displacement relationships; (b) force-velocity relationships.

Figure 17. Comparison between predicted values and real values (80 Hz-1.5 A): (a) force-displacement relationships; (b) force-velocity relationships.
force is in perfect agreement with the experimental force. Another point to demonstrate the effectiveness of the network is that the MSE between the predicted and experimental forces is calculated as 0.0867, which is small enough. Figure 14(b) shows the error distribution of training results with the range of $[-1.168, 1.125]$. It is apparent that the error range of the predicted forces is small and most of them are mainly concentrated in the vicinity of zero. According to these figures, it can be known that the trained NARX network can well depict the force responses of the MRE isolator under different loading conditions.

The comparisons of experimental and predicted results about force-displacement and force-velocity loops under different loading conditions are given in figures 15–17. Figure 15 shows the comparisons of force-displacement/velocity relationships with 0.045 mm amplitude and 60 Hz frequency excitations under four current levels (0, 0.5, 1, and 1.5 A). Figure 16 shows the hysteresis loops of the isolator with 0.09 mm amplitude and four frequencies (40, 60, 80, and 100 Hz) excitations under the current of 1 A. Figure 17 shows the relationships of force-displacement/velocity under 80 Hz, three amplitudes (0.045, 0.09 and 0.13 mm) excitations with current of 1.5 A.

It is known clearly from figures 15–17 that the proposed NARX model not only can well describe the relationship between force and displacement, but also well portray the nonlinearity between force and velocity. Therefore, the effectiveness of the trained NARX network to approximate the hysteretic behaviors is proved.

3.3. Model validation

Although the approximation capability of the NARX network for training data is proved well, the capacity of generalization is more acceptable for evaluating the performance of the network. Some new data in testing
experiments are used to validate the performance of the trained network. In addition, in order to prove the generalization capacity and accuracy of NARX network, BP network is employed to compare with NARX network. BP network is a multilayer feedforward network without internal memory, and it is trained by error back propagation algorithm. It is worth noting that the generalization capacity comparison of the two networks is carried out under the same experimental data, and the structure of BP network is also determined based on the same training error as the NARX to ensure fairness.

The comparison of the predicted force between NARX network and BP network is shown in figure 18. And the error distribution and regression analysis of the two networks are shown in figures 19 and 20, respectively. It can be known from the figures:

1. The MSE of NARX network is 0.2053, it is smaller than 3.0187 of BP network. Especially, it is seen clearly that there exist obvious errors in peak value regions of BP network in figure 18.
2. The error range of NARX network lies in $[-3.24, 2.445]$, which is smaller than $[-9.713, 9.723]$ of BP network, and the error values of the former network are also concentrated in the vicinity of zero.
3. The regression value R is 0.99923 and 0.98843 in the NARX network and BP network, respectively. The former has a better fit with the experimental data.

So, the NARX network performs a better estimated capacity for the model of MRE isolator than BP network. The effectiveness and superiority of NARX network are further demonstrated.

4. Robustness analysis of NARX network

It has been known that noise disturbance in measured inputs and output is unavoidable in real system. Therefore, it is of practical importance to study the robustness of NARX network model. The test inputs data (0.045 mm amplitude, 40 Hz frequency and four current levels) under disturbance of Gaussian white noise are chosen to validate the robustness of the proposed network. Meanwhile, the robustness comparison between NARX network and BP network is also carried out to demonstrate the resist disturbance capacity of NARX network.
4.1. Displacement signal with noise disturbance

It is assumed that there is 20 dB signal-to-noise ratio Gaussian white noise disturbance in displacement signal of each cycle. The displacement with noise is shown in figure 21. The MSE between the displacement signals with noise and without noise is 2.0262e-6.

Based on the above trained NARX network and BP network, the predicted and experimental force responses are illustrated in figure 22. The error distribution and regression value of the two networks are shown in figures 23 and 24, respectively. The performance indices for the two network models are listed in table 3.

It can be seen from figures 22–24 and table 3 that the noise in displacement signal has little influence on the approximation performance of NARX network, but produces the larger error in BP network.

4.2. Current signal with noise disturbance

Figure 25 exhibits the current signal with Gaussian white noise of 20 dB signal-to-noise ratio. The MSE between the current signals with noise and without noise is 0.0022. The test results with NARX network and BP network are shown in figures 26–28. Table 4 gives the performance indices for the two networks. It is noticeable that the disturbance in current signal has little effect on the prediction capacity of NARX network.

In the cases of the displacement and current signal with noise disturbance, the prediction forces with NARX network are always approximate the target values well and there are no singular points in the force responses. Therefore, it can be concluded that the proposed NARX network model not only provides better approximation capability but also has stronger robustness.
5. Conclusions

In this paper, the NARX network is developed to identify the model of MRE mixed mode isolator. Different from the training data generated by the ideal model, the training data of NARX is from experimental test for MRE isolator. The training and generalization results of the network show that the proposed network with 11-7-1 neurons and 2 input delays can approximate the isolator’s model well, and is more accurate than BP network by comparing MSE, error range and regression value. The results under input signals (current and displacement) with Gaussian white noise show that NARX network is stronger robustness than BP network. The analysis for robustness of the network with noise disturbance is benefit to improve the control results of MRE isolation system. It is worth mentioned that NARX network is implemented easily for model identification and on-line control of MRE isolator, which can be used for other semi-active isolators with nonlinear hysteresis and noise disturbance.

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Appendix

Based on the theory of viscoelastic materials and measured force-displacement loop of the isolator, the parameters of effective stiffness and equivalent damping can be identified to
Figure 26. Comparison of force between NARX network and BP network under current signal with noise: (a) NARX; (b) BP.

Figure 27. Comparison of error distribution between NARX network and BP network under current signal with noise: (a) NARX; (b) BP.

Figure 28. Comparison of regression between NARX network and BP network under current signal with noise: (a) NARX; (b) BP.
evaluate the performance of the isolator [28]. Figure A1 shows a sketch of force-displacement loop, in which $F_{d_{\text{max}}}$ and $F_{d_{\text{min}}}$ are the forces at the maximum and minimum displacements, respectively, and $X_{\text{max}}$ and $X_{\text{min}}$ are the maximum and minimum displacements, respectively. The effective stiffness $k_{\text{eff}}$ of the isolator follows

$$k_{\text{eff}} = \frac{F_{d_{\text{max}}} - F_{d_{\text{min}}}}{X_{\text{max}} - X_{\text{min}}}$$

The equivalent damping coefficient $c$ of the isolator can be obtained from

$$c = \frac{\Delta W}{2\pi f X^2}$$

where $\Delta W$ is the area of the force-displacement loop, $f$ and $X$ are the loading frequency and maximum loading displacement, respectively.

**References**


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