Unsteady analysis for oscillatory flow of magnetorheological fluid dampers based on Bingham plastic and Herschel–Bulkley models

Miao Yu¹, Siqi Wang¹, Jie Fu² and Youxiang Peng²

Abstract
It is known that a quasi-static analysis without considering fluid inertia is usually used for magnetorheological damper design. However, fluid inertia terms need to be incorporated into the governing Navier–Stokes equation in practical application for oscillatory or unsteady fluid flow. In this article, an unsteady flow of magnetorheological fluids within flow mode magnetorheological dampers was theoretically investigated under sinusoidal displacement excitation. Based on the governing Navier–Stokes equation, incorporating the boundary and initial conditions, central numerical velocity solutions of magnetorheological fluids within magnetorheological damping channel were developed and used to confirm the damping force using both Bingham plastic and Herschel–Bulkley models of magnetorheological fluids. To simplify the governing equation of Herschel–Bulkley flow, an approximation method replacing Herschel–Bulkley model velocity with Bingham plastic model velocity during calculating effective viscosity shear rate of Herschel–Bulkley model is proposed and used. For given magnetorheological fluid properties and flow mode magnetorheological damper geometry, under different yield stresses, sinusoidal excitation frequencies, and power-law exponent of Herschel–Bulkley model, the effects of magnetorheological fluid inertia terms on work and characteristic diagrams of magnetorheological dampers are discussed.

Keywords
Magnetorheological, unsteady flow, plug thickness, fluid inertia

Introduction
Magnetorheological (MR) fluids have attracted a lot of interests in the past decade due to their rheological characters, which can make the mechanic properties of MR devices to change with controllable electronic system (Jolly et al., 1999). MR dampers are a type of popular MR devices, in which the working space of MR fluids is usually annular gap with magnetic field activated. When MR fluids are forced to flow through the annular space from inlet to outlet, a controllable pressure drop restricted by the flow resistance of the MR fluid in the annular gap can be obtained with changing MR fluid properties by adjusting the magnetic field strength via the electronic system.

To simplify the analysis, the behavior of MR dampers is usually studied with an approximate parallel-plate geometry model instead of the actual annular damper channel when the annular gap width is relatively smaller than the piston head radius and axial length (Hong et al., 2008a, 2008b; Lee and Wereley, 1999; Phillips, 1969; Wereley and Li, 1998; Yang et al., 2002). In addition, based on the governing Navier–Stokes (NS) equations, quasi-static models without considering the fluid inertia are usually and readily used to analyze the MR fluid motion within MR dampers for initial design or performance evaluation of MR/electrorheological (ER) dampers, but the models cannot sufficiently describe a nonlinear behavior of MR dampers under dynamic loading (Li et al., 2000; Makris et al., 1996; Pang et al., 1998; Sapinski, 2002; Snyder et al., 2001; Spencer Jr. et al., 1997; Stanway et al., 1987). The fluid inertia terms should be incorporated into the governing NS equations for oscillatory or unsteady flow (Wereley and Pang, 1998), especially for the impact and...
shock loadings, such as landing gear system (Choi and Wereley, 2003) and gun recoil system (Ahmadian and Poynor, 2001). In addition, under the sinusoidal excitation, inertial effect on the unsteady flow of MR fluid is not significant in the very low range of the oscillatory frequency, but when the oscillatory frequency is higher, the effect is obvious (Li et al., 2000).

Obviously, it is highly desirable to study an unsteady flow of MR fluid within MR dampers to sufficiently describe the nonlinear behavior of MR dampers under dynamic loading, especially for the unsteady flow of MR fluid with Herschel–Bulkley (HB) model that accounts for shear-thinning/shear-thickening behavior at higher strain rates. Chen et al. (2004) solved the velocity profile and pressure gradient of the unsteady state unidirectional flow of Bingham fluid between parallel plates by the Laplace transform method. Based on the proposed unsteady flow solution of an ER fluid flow, Nguyen and Choi (2009) developed a dynamic modeling of the ER damper considering the unsteady behaviors of ER fluid flow through the annular duct of the damper for parameter identification; in addition, experiments were conducted to validate the proposed model and showed that the effect of unsteady behavior is significant and should be taken into account at a high frequency.

The main objective of this article is to theoretically analyze unsteady flow of MR fluids with Bingham plastic (BP) and HB model within flow mode MR dampers by means of numerical velocity solutions of partial differential equations (PDE). In section “Simplified mathematical model of the flow mode MR dampers,” a simplified mathematical model of flow mode MR dampers is developed. The governing equation and the boundary and initial conditions are presented and incorporated into the PDE. In section “Velocity solutions,” the numerical velocity solutions of damping channel for BP and HB models are all derived. In section “Work and characteristic diagrams of MR dampers,” the damping force is confirmed, and MR fluid inertia effects on work and characteristic diagrams are discussed. Finally, conclusions are drawn in section “Conclusion.”

Simplified mathematical model of the flow mode MR dampers

Governing equations

In this section, a flow mode MR damper (Zhu et al., 2012) is investigated, as shown in Figure 1; when external load acts on the piston rod, MR fluids will be forced to flow against flow resistance from the annular

![Figure 1. Schematic diagram of the flow mode MR damper.](image)

MR: magnetorheological.
damping channel. It is known that the controllable damping force can be obtained for the rheological characters of MR fluids flowing through the annular damping channel. When the damping channel width \((d)\) is smaller than the damping channel inner radius \((R_1)\) and effective length \((L)\), the behavior of MR dampers is often studied with an approximate parallel-plate geometry model instead of the actual annular damping channel, and the pressure gradient between piston head sides \(\Delta P(t)\) is assumed to evenly distribute along the damping channel length (Wereley and Pang, 1998). According to NS equations, for the parallel-plate model, a one-dimensional (1D) governing equation can be written as

\[
\rho \frac{\partial u(y, t)}{\partial t} = -\frac{\Delta P(t)}{L} + \frac{\partial \tau(y, t)}{\partial y}
\]

where \(\rho\) is the fluid density, \(t\) is the time variable, \(u(y, t)\) is the fluid velocity, \(\Delta P(t)\) is the pressure difference between sides of the piston head, and \(\tau(y, t)\) is the shear stress.

**Constitutive equations**

In this article, both BP model (Stanway et al., 1996; Wereley and Pang, 1998) and HB model (Lee and Wereley, 1999; Wang and Gordaninejad, 1999) are used to analyze the unsteady flow behaviors of MR fluid. The constitutive equations of the two models are given as follows

**BP model**

\[
\begin{align*}
\tau &= \tau_y \cdot \text{sgn}(\dot{\gamma}) + \eta \dot{\gamma}, \quad |\tau| > \tau_y, \\
\dot{\gamma} &= 0, \quad |\tau| < \tau_y
\end{align*}
\]

**HB model**

\[
\begin{align*}
\tau &= \tau_y \cdot \text{sgn}(\dot{\gamma}) + \kappa (|\dot{\gamma}|^m)^{1/m}, \quad |\tau| > \tau_y, \\
\dot{\gamma} &= 0, \quad |\tau| < \tau_y
\end{align*}
\]

where \(\eta\) is zero-field viscosity of MR fluids, \(\tau_y\) is the critical yield stress, and \(\dot{\gamma} = (du(y, t)/dt)\) is the rate of stress strain.

**Boundary conditions**

Figure 2 shows a typical velocity profile of MR fluids flowing through damping channel of flow mode MR dampers. There are two coordinates, the absolute coordinate system \((x, r)\) and the parallel-plate gap coordinate system \((x, y)\); \(x\) is taken as the centerline direction in the two parallel plates, \(y\) is the coordinate normal to the plate, and \(r\) denotes the coordinate along the piston radii. Assuming that the piston with velocity \(V_0\) moves in the negative direction of \(x\)-axis, the velocity profile in the positive direction of \(x\)-axis is formed by the pressure gradient between the sides of the piston head.

**Figure 2.** Typical MR fluid velocity profile in the annular damping channel of flow mode MR dampers. MR: magnetorheological.
Table 1. Boundary conditions of annular damping channel for flow mode MR dampers

<table>
<thead>
<tr>
<th>Regions</th>
<th>Region 1 ((0 \leq y &lt; y_{po}))</th>
<th>Region 2 ((y_{po} \leq y &lt; y_{po}))</th>
<th>Region 3 ((y_{po} \leq y \leq d))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boundary conditions</td>
<td>(u(0, t) = 0) and (\tau(y_{po}, t) = \tau_y) and (u_t(y_{po}, t) = 0)</td>
<td>(u(0, t) = 0) and (\tau(y_{po}, t) = -\tau_y) and (u_t(y_{po}, t) = 0)</td>
<td>(u(d, t) = 0)</td>
</tr>
</tbody>
</table>

MR: magnetorheological.

According to the velocity profile, boundary of parallel-plate gap width \((d)\), inner boundary of the plug region \((y_{pi})\), and outer boundary of the plug region \((y_{po})\) divide parallel-plate gap into three regions, region 1 (post-yield) \((0 \leq y < y_{pi})\), region 2 (preyield) \((y_{pi} \leq y < y_{po})\), and region 3 (postyield) \((y_{po} \leq y \leq d)\), and the boundary conditions of the regions are listed in Table 1.

Mathematical equation

The sinusoidal displacement excitation is usually applied on MR dampers to characterize their properties, such as work diagram (force vs displacement profile) and characteristic diagrams (force vs velocity profile); therefore, in this article, the sinusoidal displacement excitation is used to load on the piston of MR dampers, and the sinusoidal displacement excitation function is

\[
x(t) = A \sin(\omega t)
\]

where \(x(t)\) is the sinusoidal displacement excitation, and \(A\) and \(\omega\) are the sinusoidal displacement excitation amplitude and angular velocity, respectively. Based on the sinusoidal displacement excitation equation, assuming that pressure gradient between piston head sides \(\Delta P(t)\) is evenly distributed along the damping channel length \((L)\), the pressure gradient \(\Delta P(t)\) satisfies the following equation

\[
\frac{\Delta P(t)}{L} = \rho p(t) \cos(\omega t)
\]

where \(p(t) = \sum_{i=0}^{n} a_i t^i\), a polynomial of time variable; \(a_i\) are certain constants. According to \(p(t)\), to obtain a precise velocity solution, there must be at least \(n\) equations to solve, which is relatively complex. To simply the analysis, the \(p(t_i)\) can be seen as a constant \(B_i\) for every time point \(t_i\) under sinusoidal displacement excitation, namely

\[
\frac{\Delta P(t_i)}{L} = \rho B_i \cos(\omega t_i)
\]

where \(t_i = iT/n\), where \(T\) is the sinusoidal displacement excitation cycle, \(0 \leq i \leq n\). It is known that there is a determinate \(B\) for certain \(t \in [0, T]\). In addition, assuming that the initial condition is satisfied using the following equation

\[
u(y, 0) = -u_y(y, \frac{T}{2})
\]

Based on the above equations, the mathematical method (PDE) describing the MR fluids flowing through damping channel can be developed for every region incorporating the governing equation, boundary conditions, and initial condition. The velocity of MR fluid for each region satisfies an independent PDE, namely,

In region 1

\[
\begin{align*}
\frac{\partial u_1(y, t)}{\partial t} & = \nu \cdot \frac{\partial u_{1yy}(y, t)}{\partial y} + B \cos(\omega t) \\
\nu u_1(0, t) & = 0, u_{1x}(y_{pi}, t) = 0 \\
\nu u_1(y, 0) & = -u_1(y, \frac{T}{2})
\end{align*}
\]

In region 2

\[
\begin{align*}
\frac{\partial u_2(y, t)}{\partial t} & = \nu \cdot \frac{\partial u_{2yy}(y, t)}{\partial y} + B \cos(\omega t) \\
\nu u_2(0, t) & = 0, u_{2x}(y_{po}, t) = 0 \\
\nu u_2(y, 0) & = -u_2(y, \frac{T}{2})
\end{align*}
\]

In region 3

\[
\begin{align*}
\frac{\partial u_3(y, t)}{\partial t} & = \nu \cdot \frac{\partial u_{3yy}(y, t)}{\partial y} + B \cos(\omega t) \\
\nu u_3(d, t) & = 0, \nu u_{3x}(y_{po}, t) = 0 \\
\nu u_3(y, 0) & = -u_3(y, \frac{T}{2})
\end{align*}
\]

where \(\nu = \frac{\eta}{\rho}\), \(u_y(y, t) = \frac{\partial u(y, t)}{\partial t}\), \(u_{xy}(y, t) = \frac{\partial^2 u(y, t)}{\partial y^2}\), and \(u_{yy}(y, t) = \frac{\partial^2 u(y, t)}{\partial y^2}\).

Velocity solutions

Velocity solutions for BP model

The PDE is solved in variable separation method (see Appendix 1). Here, coefficient \(K_{B1}\) in region 1 and coefficient \(K_{B3}\) in region 3 for BP model are defined as

\[
K_{B1} = \left[\frac{(2n-1)\pi}{2y_{pi}}\right]^2 \nu (n = 1, 2, 3, \ldots)
\]

\[
K_{B3} = \left[\frac{(2n-1)\pi}{2(d-y_{po})}\right]^2 \nu
\]
The velocity solutions for BP model in each region are given as follows

\[ u_{B1}(y, t) = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi(K_{B1}^2 + \omega^2)} \times [K_{B1} \cos \omega t + \omega \sin \omega t]\sin(2n-1)\pi y \]

\[ u_{B2}(y, t) = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi(K_{B1}^2 + \omega^2)} \times [K_{B1} \cos \omega t + \omega \sin \omega t] \]

\[ u_{B3}(y, t) = \sum_{n=1}^{\infty} \frac{4B}{(2n-1)\pi(K_{B3}^2 + \omega^2)} \times [K_{B3} \cos \omega t + \omega \sin \omega t]\sin(2n-1)\pi(y - d) \]

\[ \frac{2(d - y_{po})}{2y_{pi}} \]

(13)

**Velocity solutions for HB model**

The HB model can be seen as the BP model by the relationship listed in equation (4). To simplify the calculation, the solutions of BP model known are used to approximate calculate the rate of strain of the HB model in equation (4) as a known quantity; hence, the rates of strain in region 1 \( \gamma_{H1} \) and region 3 \( \gamma_{H3} \) can be described as

\[ \gamma_{H1} = \frac{du_{B1}(y, t)}{dy} = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi(K_{B1}^2 + \omega^2)} \times [K_{B1} \cos \omega t + \omega \sin \omega t] \sin(2n-1)\pi y \]

\[ \frac{2y_{pi}}{2y_{pi}} \cos \frac{2(2n-1)\pi y}{2y_{pi}} \]

\[ \gamma_{H3} = \frac{du_{B3}(y, t)}{dy} = \sum_{n=1}^{\infty} \frac{4B}{(2n-1)\pi(K_{B3}^2 + \omega^2)} \times [K_{B3} \cos \omega t + \omega \sin \omega t] \sin \frac{2n-1)\pi(y - d)}{2(d - y_{po})} \]

(14)

Based on the known rates of strain in the equation (14), the flow velocity solutions procedure of MR fluid with HB model is similar to that of MR fluid with BP model, and the flow velocity solutions of MR fluid with HB model can be obtained using coefficients \( K_{H1} \) and \( K_{H3} \) instead of coefficients \( K_{B1} \) and \( K_{B3} \) respectively. The coefficients \( K_{H1} \) and \( K_{H3} \) are given as follows

\[ K_{H1} = \frac{(2n-1)\pi}{2y_{pi}} \frac{\kappa(\gamma_{H1})^{m-1}}{\rho} \]

\[ K_{H3} = \frac{(2n-1)\pi}{2(d - y_{po})} \frac{\kappa(\gamma_{H3})^{m-1}}{\rho} \]

(15)

The velocity solutions of HB model are given as follows

\[ u_{H1}(y, t) = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi(K_{H1}^2 + \omega^2)} \times [K_{H1} \cos \omega t + \omega \sin \omega t]\sin(2n-1)\pi y \]

\[ \frac{2y_{pi}}{2y_{pi}} \cos \frac{2(2n-1)\pi y}{2y_{pi}} \]

\[ u_{H2}(y, t) = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi(K_{H2}^2 + \omega^2)} \times [K_{H2} \cos \omega t + \omega \sin \omega t] \]

\[ \frac{2(d - y_{po})}{2y_{pi}} \cos \frac{2n-1)\pi(y - d)}{2(d - y_{po})} \]

(16)

**Work and characteristic diagrams of MR dampers**

**Damper force**

According to continuity equation of the incompressible MR fluid flow, it is known that the total volume flux through the damping channel \( Q_c \) equals the volume flux displaced by the piston \( Q_p \), namely

\[ Q_c = Q_p = S_p V_0 \]

(17)

where \( S_p \) is the effective cross section of piston head and \( V_0 \) is the velocity of the piston. The total volume flux consists of the volume fluxes in each region obtained by velocity-annular gap width integral, namely

\[ Q_c = Q_1 + Q_2 + Q_3 = b \int_0^{y_{po}} u_1(y, t)dy + b \int_{y_{po}}^{y_{pi}} u_2(y, t)dy + b \int_{y_{pi}}^{d} u_3(y, t)dy \]

\[ = b \sum_{n=1}^{\infty} \frac{4B}{(2n-1)\pi(K_{H1}^2 + \omega^2)} [K_{H1} \cos \omega t + \omega \sin \omega t] \sin \frac{2y_{pi}}{2(2n-1)\pi} \]

\[ + \sum_{n=1}^{\infty} \frac{4B}{(2n-1)\pi(K_{H3}^2 + \omega^2)} [K_{H3} \sin \omega t - \omega \cos \omega t] \sin \frac{2(d - y_{po})}{2(2n-1)\pi} \]

(18)

where \( b \) is the average perimeter of the annular damping channel; \( u_1(y, t) \) is general term for \( u_{B1}(y, t) \) and \( u_{H1}(y, t) \); \( u_2(y, t) \) is general term for \( u_{B2}(y, t) \) and \( u_{H2}(y, t) \); \( u_3(y, t) \) is general term for \( u_{B3}(y, t) \) and \( u_{H3}(y, t) \).
u_{H3}(y, t); coefficient $K_1$ is general term for coefficients $K_{81}$ and $K_{H1}$; and coefficient $K_3$ is general term for coefficients $K_{83}$ and $K_{H3}$.

The plug flow region is rigid, and the total yield stress ($\tau(y_{pi}) - \tau(y_{po}) = 2\tau_p$) linearly varies along the plug thickness. Based on the force equilibrium of plug region 2, as shown in Figure 3, the plug thickness can be obtained and rewritten as

$$y_{po} - y_{pi} = 2\tau_p \left| \frac{\Delta P(t)}{L} - \rho \frac{\partial u_2(y, t)}{\partial t} \right|^{-1}$$  \hspace{1cm} (19)

In addition, according to the boundary condition in region 2 and plug velocity solved in equation (13) or (16), the system of equations can be obtained as follows

$$y_{po} + y_{pi} = d$$  \hspace{1cm} (20)

Substituting equation (18) into equation (17) and equation (7), (13), or (16) into equation (19), according to equations (17), (19), and (20), the numerical solution of pressure drop $\Delta P(t)$ can be worked out, and at the same time, the damping force $F(t)$ can be obtained by the following equation

$$F(t) = \Delta P(t)S_p$$  \hspace{1cm} (21)

### Dynamic performance of work and characteristic diagram

In this study, both BP and HB models are discussed for dynamic performance of work and characteristic diagram of MR dampers. Under certain sinusoidal displacement excitations, dynamic damping forces are obtained by numerical calculation with different excitation frequencies, yield stresses, and power-law exponent of HB model, and the damping forces driven from the quasi-static analysis models of BP and HB (Wereley and Li, 1998; Yang et al., 2002) are used to compare with dynamic damping forces. According to the damping forces, sinusoidal displacement excitation, and velocity, dynamic performances of work and characteristic diagram of MR dampers are investigated. Referring to design methods of ER or MR dampers in the literatures (Nguyen, et al., 2007; Rosenfield and Wereley, 2004; Wang et al., 2009), configuration parameters of flow mode MR dampers are provided and used for unsteady analysis, as listed in Table 2.

### Frequency dependence

The relationships of MR damper response between force versus displacement and force versus velocity with yield stress (20 kPa) and various frequencies (5, 10, 20, and 30 Hz) are shown in Figure 4(a) and (b), respectively. It can be seen from Figure 4(a) that all hysteresis work loops of force versus displacement are approximately elliptical. In addition, the damping force increases with the increase in frequency at any moment. The BP-like behavior of MR

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**Table 2. Simulation parameters**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinusoidal displacement excitation amplitude ($A$)</td>
<td>0.02 m</td>
</tr>
<tr>
<td>MR fluid yield stress ($\tau_p$)</td>
<td>10, 20, and 30 kPa</td>
</tr>
<tr>
<td>Sinusoidal excitation frequency ($f$)</td>
<td>5, 10, 20, and 30 Hz</td>
</tr>
<tr>
<td>Damping channel length ($L$)</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Damping channel inner radius ($R_1$)</td>
<td>0.0185 m</td>
</tr>
<tr>
<td>Damping channel outer radius ($R_2$)</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Piston rod radius of MR dampers ($R_{pr}$)</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Average perimeter of the damping channel ($b$)</td>
<td>0.1209 m</td>
</tr>
<tr>
<td>Damping channel width ($d$)</td>
<td>0.0015 m</td>
</tr>
<tr>
<td>Mass density of MR fluids ($\rho$)</td>
<td>$2.650 \times 10^3$ kg/m$^3$</td>
</tr>
<tr>
<td>Zero-field viscosity of MR fluid ($\eta$)</td>
<td>1.000 Pa s</td>
</tr>
</tbody>
</table>

MR: magnetorheological.
Fluids can be seen in MR damper response between damping force versus velocity, as shown in Figure 4(b). For the same yield stress and different frequencies, there is the same yield force that can be seen as the intercept of force versus velocity curves projected back to the force axis, and the higher the frequency, the larger the peak damper force. It can be seen from Figure 4 that the relationships of MR damper response between force versus displacement and force versus velocity show good agreement between the quasi-static analysis results and the unsteady analysis results at low frequency, but the nonlinear behavior of the force versus velocity loop is more obvious at high frequency. In addition, to study the effect of the MR fluid unsteady...
flow on the velocity, Figure 4(c) shows the MR fluid velocity profile in the MR valve with yield stress (20 kPa) and various frequencies (5, 10, 20, and 30 Hz) at the time of 3/16 sine cycles. The plug flow with the constant velocity is in the central region of the MR valve and developed due to the yield stress of MR fluid. In postyield region, the flow velocity profile is confirmed by equation (13). The velocity profile is affected by the MR fluid inertia; hence, it can be seen that in Figure 4(c), the velocity profile of the unsteady analysis is higher than that of quasi-static analysis. In addition, the higher the sinusoidal excitation frequencies, more significant the effect of MR fluid inertia on the MR fluid flow.

**Yield stress (magnetic field) dependence.** It is known that MR dampers work in the preyield region when the damper force is less than fixed yield force, and MR dampers operate in the postyield region when the damping force increases beyond fixed yield force. The BP model can accurately describe the postyield behavior of MR dampers. Figure 5(a) and (b) shows the relationships of force versus displacement and force versus velocity with different yield stresses (10, 20, and 30 kPa) at fixed frequency (30 Hz), respectively. It is known that yield force is controlled by the corresponding applied current (magnetic field). It is shown in Figure 5(a) that the higher the yield stress, the larger the yield force, and MR dampers work more in the postyield region with decreasing yield stress (magnetic field). Figure 5(b) shows the BP-like behavior of MR fluids. In addition, based on the results between quasi-static analysis and the unsteady analysis, it can be seen that the effect of MR fluid yield stress on the dynamic damping force of MR dampers is not significant.

**Power-law exponent of HB model dependence.** In this study, the HB model is also used to investigate the dynamic performance of work and characteristic diagram of MR dampers. To compare with the BP model, the consistency index \(k\) in equations (3) and (4) is assumed to equal zero-field viscosity of BP model \(\eta\). The power-law exponent of HB model \(m\) represents shear-thinning \((m < 1)\) and shear-thickening \((m > 1)\) behaviors of MR fluids. The values of \(m\), 0.9 and 1.0, are discussed since most of the MR fluids show the shear-thinning behaviors. Figure 6(a) shows the relationship of force versus displacement
for HB model with yield stress (20 kPa) and frequency (30 Hz). The damping force decreases with decreasing value of $m$, whereas viscosity of MR fluid decreases with its thinning behavior. The HB-like behavior of MR dampers from the relationships of force versus velocity is shown in Figure 6(b). According to quasi-static and the unsteady analysis, the effect of shear-thinning behavior of MR fluid on the dynamic damping force of MR dampers is also not significant.

**Experimental test**

To validate the theoretical analysis, in this article, a double-ended MR fluid damper with flow mode is manufactured and tested using a MTS 831 elastomer test system, as shown in Figure 7. The MR fluid control

![Experimental setup](image)

**Figure 7.** Experimental setup.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Damping channel outer radius</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Damping channel inner radius</td>
<td>0.0185 m</td>
</tr>
<tr>
<td>Piston rod radius of MR dampers</td>
<td>0.01 m</td>
</tr>
<tr>
<td>Damping channel length</td>
<td>0.02 m</td>
</tr>
<tr>
<td>Damping channel width</td>
<td>0.0015 m</td>
</tr>
</tbody>
</table>

MR: magnetorheological.
valve is assembled in the piston, and its structure parameters are listed in Table 3. The MR damper with the input current of 1 A was subjected to sinusoidal excitation with the amplitude of 0.01 m. Figure 8(a) and (b) shows the relationships of force versus displacement and force versus velocity at a fixed frequency of 10 Hz, respectively. The results show that there is a little difference between the quasi-static and unsteady simulated results, and unsteady simulated results agree well with the tested results.

To well show the effect of MR fluid unsteady flow on the damping force, a higher frequency of 20 Hz is considered. Figure 9(a) and (b) shows the relationships of force versus displacement and force versus velocity at a fixed frequency of 20 Hz, respectively. The results show that there is a significant difference between the quasi-static and unsteady simulated results, and unsteady simulated results also agree well with the tested results. From Figures 8 and 9, it is known that the analysis method of MR fluid unsteady flow can be used to precisely predict the MR damping force in comparison with quasi-static analysis method. In addition, it is also seen that the effect of MR fluid unsteady behavior is significant and should be taken into account at the high frequency.

**Conclusion**

In this article, an unsteady analysis method was presented for oscillatory flow of MR fluid within flow mode MR damper under sinusoidal displacement excitation. According to the governing NS equations and the boundary and initial conditions, numerical velocity solutions of damping channel are derived and used to confirm the damping force. Based on the damping force, sinusoidal displacement excitation, and velocity, work and characteristic diagrams of BP and HB model were discussed. The relationships of force versus velocity well show BP or HB model-like behaviors. From the results between quasi-static analysis and the unsteady analysis, it is seen that unsteady flow effect on the dynamic damping force is significant for sinusoidal excitation frequencies, but the effects due to fluid yield stress and power-law exponent of HB model on the dynamic damping force are not significant. Compared with quasi-static analysis model, the unsteady analysis method can well describe the nonlinear behavior of MR dampers under sinusoidal loading, and the nonlinear behavior of MR dampers is more obvious at high frequency. According to the demand, either BP or HB

![Figure 8.](image-url)

Figure 8. Relationships of (a) force versus displacement and (b) force versus velocity of MR damper with the input current of 1 A ($f = 10$ Hz).
model could be chosen to analyze unsteady flow of MR fluids theoretically; in addition, the theoretical analysis with BP model is simpler than that with HB model, for unsteady velocity to HB model is confirmed with the help of unsteady velocity to BP model.

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**References**


**Figure 9.** Relationships of (a) force versus displacement and (b) force versus velocity of MR damper with the input current of 1 A ($f = 20$ Hz).


Appendix 1

Solving PDE in variable separation method

PDE in region 1 is used as an example

\[
\begin{aligned}
\begin{cases}
   u_{1,H}(y,t) = n \cdot v \cdot u_{1y}\(y,t) + B \cos(\omega t) \\
   u_{1}(0,t) = 0, u_{1y}(y_{pi},t) = 0 \\
   u_{1}(y,0) = -u_{1}(\frac{y}{2})
\end{cases}
\end{aligned}
\]  

(22)

Assume

\[
\begin{aligned}
   u_{1}(y,t) = Y(y)T(t)
\end{aligned}
\]  

(23)

where \( Y(y) \) and \( T(t) \) are the functions of variables \( y \) and \( t \), respectively. First, according to the given boundary conditions confirm the intrinsic value and intrinsic function of homogeneous equation \( u_{1}(y,t) = \nu \cdot u_{1y}(y,t) \)

\[
\begin{aligned}
\begin{cases}
   Y_{yy} + \lambda Y = 0 \\
   Y(0) = 0, Y_{y}(y_{pi}) = 0
\end{cases}
\end{aligned}
\]  

(24)

According to Sturm–Liouville equation, \( \lambda > 0 \), assuming \( \lambda = \mu^{2} (\mu > 0) \), the general solution is given as

\[
\begin{aligned}
   Y(y) = \alpha \cos \mu y + \beta \sin \mu y
\end{aligned}
\]  

(25)

where \( \alpha \) and \( \beta \) are constants, and \( \alpha \beta \neq 0 \). Substituting equation (25) into equation (24), we get

\[
\begin{aligned}
\begin{cases}
   \alpha = 0 \\
   \beta \cos \mu y_{pi} = 0
\end{cases}
\end{aligned}
\]  

(26)

where \( \beta \neq 0 \), so intrinsic value is \( \lambda_{n} = \mu_{n}^{2} = [(2n-1)\pi/(2y_{pi})]^{2} \), and the corresponding intrinsic function of \( Y_{n}(y) \) is \( \sin \mu_{n} y \).

Assume

\[
B \cos(\omega t) = \sum f_{n}(t) Y_{n}(y)
\]  

(27)

where \( f_{n}(t) \) is

\[
f_{n}(t) = \frac{2}{y_{pi}} \int_{0}^{y_{pi}} B \cos(\omega t) \sin \mu_{n} y dy
\]  

(28)

Substituting intrinsic value \( \lambda_{n} \) into the following equation

\[
T_{n}(t) + \lambda_{n} T_{n}(t) = f_{n}(t)
\]  

(29)

We get

\[
T_{n}(t) = \frac{-4B}{(2n-1)\pi [K_{B1} + \omega^{2}]} \times [K_{B1} \sin \omega t - \omega \cos \omega t + C \exp^{-K_{B1}t}]
\]  

(30)

where \( K_{B1} = [(2n-1)\pi/(2y_{pi})]^{2} \), we get

\[
u(y,t) = \sum_{n=1}^{\infty} u_{1n}(y,t) = \sum_{n=1}^{\infty} Y_{n}(y)T_{n}(t)
\]  

(31)

With the initial condition \( u_{1}(y,0) = -u_{1}(T/2) \), the fluid velocity for BP model in region 1 is

\[
u_{B1}(y,t) = \sum_{n=1}^{\infty} \frac{-4B}{(2n-1)\pi [K_{B1} + \omega^{2}]} \times [K_{B1} \cos \omega t + \omega \sin \omega t] \sin\left(\frac{(2n-1)\pi y}{2y_{pi}}\right)
\]  

(32)

Similarly, the fluid velocity for BP model in region 3 can be obtained, as shown in equation (13).