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Neural network compensation of semi-active control for magneto-rheological suspension with time delay uncertainty

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Abstract
This study presents a new intelligent control method, human-simulated intelligent control (HSIC) based on the sensory motor intelligent schema (SMIS), for a magneto-rheological (MR) suspension system considering the time delay uncertainty of MR dampers. After formulating the full car dynamic model featuring four MR dampers, the HSIC based on eight SMIS is derived. A neural network model is proposed to compensate for the uncertain time delay of the MR dampers. The HSIC based on SMIS is then experimentally realized for the manufactured full vehicle MR suspension system on the basis of the dSPACE platform. Its performance is evaluated and compared under various road conditions and presented in both time and frequency domains. The results show that significant gains are made in the improvement of vehicle performance. Results include a reduction of over 35% in the acceleration peak-to-peak value of a sprung mass over a bumpy road and a reduction of over 24% in the root-mean-square (RMS) sprung mass acceleration over a random road as compared to passive suspension with typical original equipment (OE) shock absorbers. In addition, the semi-active full vehicle system via HSIC based on SMIS provides better isolation than that via the original HSIC, which can avoid the effect of the time delay uncertainty of the MR dampers.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

Automotive ride quality and handling performance remain a challenging design for modern passive automobile suspension systems. Despite extensive published research outlining the benefits of active vehicle suspension in addressing this tradeoff, the cost and complexity of these systems prohibit commercial adoption. Consequently, the research on vibration control using semi-active suspensions has increased greatly since semi-active suspensions can provide performance benefits over passive suspensions without the cost and complexity associated with fully active systems. Among those semi-active suspensions, a very attractive semi-active suspension featuring a magneto-rheological (MR) damper has been proposed by many investigators in the last decade [1–9], because of its...
fast response characteristic to magnetic fields and hence wide control bandwidth and compact size.

The time delay of a MR semi-active suspension system from sensing the structural states to the reaction of the MR damper is an important performance property in practical use. It is noted that this property is controlled by three different time delays, which are the time delay of the control electronics, that of the electrical part and that of the mechanical part of the MR damper itself, respectively, in the closed control loop [10]. Generally, the time delay of the control electronics is dependent on the adoption of commercial hardware and control software, and can be further reduced by using more advanced hardware and an optimized control algorithm. Whereas some studies have shown that the responses of devices using MR fluids are not as fast as people anticipate [11, 12], it is said that the response time of the MR fluid itself is a few ms (around 1–2 ms) [13, 14]. Therefore, sometimes the entire time delay of the MR suspension system is mainly given by the response time of the electrical part and the mechanical part of the MR damper [15].

Several attempts to characterize the response time of the electrical part and the mechanical part of the MR devices are described in the literature [11, 13, 16–19]. Those studies show that the response time of MR devices is often limited, not by the response of the fluid itself, but by the limitations of the driving electronics and the inductance of the electromagnet. Reported results cover a broad range from 0.1 up to 100 ms, depending on the method applied. For example, the product bulletin of the Lord Corporation’s 180 kN MR damper states that the damper’s response time is less than 60 ms [11]. Koo et al [16, 17] measured the time delay of a commercially available damper from Lord Corporation and the result is 25 ms. Zhu [18] tested the time delay of a disc-type MR fluid damper using a step current mode and found that the time delay fell into a regime between 0.08 and 0.4 s, depending on the type of MR fluid and the rotational speed of the disc. Goncalves et al [19] investigated the time delay at high shear rates and the dwell time is as low as 0.45–0.6 ms.

In the meantime, the time delay of MR devices is an uncertain variable and has relations with many factors such as the device dimension, working conditions and measuring method. Koo et al [20] tested the response time of an MR damper under various operating currents and piston velocities. The results show that the response time of the MR damper decreases as the operating current increases for a given velocity, or as the velocity increases for a given current. The results are also supported by the authors’ work [21]. There might be some reasons for the uncertainty in time delay of an MR damper. One is the physical dimensions of the MR damper. A bigger MR damper may need a longer time to achieve the same control force of a smaller MR damper. One means to solve this problem is to use several small MR dampers instead of using a single large MR damper. The second reason may be due to the type of driving electronics, the eddy current and the leakage of the magnetic field. Naoyuki et al [22] studied the method to improve the response of a MR actuator and found that the response of the MR actuator will become slower when considering the eddy current of the yoke in the MR actuator. The third reason may be the MR fluid dynamics (the rearrangement of particles), which causes the different response times for different piston velocities [20].

Therefore, the time delay of MR devices should not be negligible in the development of semi-active control algorithms for those MR devices when pursuing high control performance. However, there are few publications considering the time delay of MR dampers during the design of the control strategy. Park et al [23] measured the time delay of an MR damper when the voltage is from on to off or from off to on and studied the robust control of semi-active vibration of a controllable seat system considering the time delay. Choi et al [24] formulated a sliding mode controller for an electro-rheological (ER) damper system subjected to the time delay of an ER damper. The control method has been proven effective through simulation under bump excitation.

In previous works [30, 31], the authors proposed a new intelligent control algorithm, human-simulated intelligent control (HSIC), for a half-car model featuring two MR dampers considering the nonlinearity and time delay of the MR dampers. The simulation and road test show that the control method can improve the ride comfort of the vehicle and avoid the effect of time delay using a compensation method. Nevertheless, the time delay of the MR damper was assumed as an unchanging value. In addition, the control strategy was formulated on the basis of a half-car dynamic model or a quarter-car model, without considering the coupling relation of the vehicle motions and four control inputs of four MR dampers.

Consequently, the main contribution of this study is to investigate the effect of the time delay uncertainty of an MR damper caused by the electrical part and the mechanical part on the vibration control performance of a passenger vehicle suspension system and its adaptive compensation method. In order to achieve this goal, an improved HSIC based on sensory motor intelligent schema (SMIC) is first proposed and formulated for the full-car model featuring four MR dampers. Then the time delay of the MR damper is empirically identified and integrated with a full-car MR suspension system. This will be followed by a derived backpropagation network to compensate for the time delay uncertainty of MR dampers. Finally, vibration control responses of the full-car MR suspension system under various control strategies subjected to bump and random excitations are numerically and experimentally evaluated.

2. HSIC based on SMIS for full vehicle featuring four MR dampers with time delay uncertainty

2.1. Full-car dynamic model considering the time delay uncertainty of MR dampers

A schematic of the HSIC system with time delay compensation is shown in figure 1. The controller consists of two parts. One is the HSIC based on SMIS, which calculates the control force according to error and the change in the error; the other is the time delay uncertainty compensation neural network (NN) model.
respectively. $I_{xx}$, $I_{yy}$, and $I_{zz}$ are the unsprung masses of the front left suspension, front right suspension, rear left suspension and rear right suspension, respectively. $I_{xx}$, $I_{yy}$ are the roll and pitch mass moments of inertia, respectively. $k_b, k_{fr}, k_{rl}, k_{rr}$ are the spring constants of the suspensions. $c_{fl}, c_{fr}, c_{rl}, c_{rr}$ are the basal damping coefficients of the four MR dampers and $F_{dfl}, F_{dfr}, F_{drl}, F_{drr}$ are the controllable damping forces of the four MR dampers. $k_{fl}, k_{fr}, k_{rl}, k_{rr}$ are the stiffness coefficients of the tires. $z_{fl}, z_{fr}, z_{rl}, z_{rr}$ are the vertical displacements of the four corners of the vehicle body and $z_{att}, z_{afl}, z_{arl}, z_{arr}$ are the vertical displacements of the unsprung mass. $z, \theta, \varphi$ are the heave displacement, pitch and roll angular displacement, respectively. $z_{rfl}, z_{rfr}, z_{rrl}, z_{rrr}$ are the road excitations. $a, b$ are the distances between the front damper and the center of gravity (C.G.) of the sprung mass and the distance between the rear damper and the C.G. of the sprung mass. $d$ denotes the width of the car body.

If new state variables are defined as

$$x = \begin{bmatrix} x_1, \ldots, x_{14} \end{bmatrix}^T = \begin{bmatrix} z, \dot{z}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z_{att} - z_{rfl}, \cdots, z_{arr} - z_{rrr} \end{bmatrix}^T.$$  

Equation (1) can be formulated in a standard state space form as

$$\dot{x} = Ax + Bu + Lw \quad (2)$$

where

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{11}/m_s & a_{12}/m_s & 0 & a_{13}/m_s & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{21}/I_{xx} & a_{22}/I_{xx} & 0 & a_{23}/I_{xx} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
a_{31}/I_{yy} & a_{32}/I_{yy} & 0 & a_{33}/I_{yy} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{fl}/m_{ufl} & 0.5dk_{fl}/m_{ufl} & 0 & -ak_{fl}/m_{ufl} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{fr}/m_{ufr} & 0.5dk_{fr}/m_{ufr} & 0 & -ak_{fr}/m_{ufr} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{rl}/m_{url} & 0.5dk_{rl}/m_{url} & 0 & -ak_{rl}/m_{url} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
k_{rr}/m_{urr} & 0.5dk_{rr}/m_{urr} & 0 & -ak_{rr}/m_{urr} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Figure 1. HSIC with time delay compensator for full-car model with four MR dampers.
\[ w = \begin{bmatrix} z_{\text{a}} \\ z_{\text{fr}} \\ z_{\text{rr}} \end{bmatrix}, \]

\[ a_{11} = -(k_{\text{a}} + k_{\text{fr}} + k_{\text{rr}} + k_{\text{fr}}), \]
\[ a_{12} = -0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{rr}}} - 0.5d_{k_{\text{fr}}}, \]
\[ a_{13} = a_{k_{\text{a}}} + a_{k_{\text{fr}}} - b_{k_{\text{a}}} - b_{k_{\text{fr}}}, \]
\[ a_{21} = -0.5d_{k_{\text{fr}}} + 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}}, \]
\[ a_{22} = 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} + 0.5d_{k_{\text{fr}}}, \]
\[ a_{23} = 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} + 0.5d_{k_{\text{fr}}}, \]
\[ a_{31} = a_{k_{\text{a}}} + a_{k_{\text{fr}}} - b_{k_{\text{a}}} - b_{k_{\text{fr}}}, \]
\[ a_{32} = 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} - 0.5d_{k_{\text{fr}}} + 0.5d_{k_{\text{fr}}}, \]
\[ a_{33} = -a_{k_{\text{a}}}^2 - a_{k_{\text{fr}}}^2 - b_{k_{\text{a}}}^2 - b_{k_{\text{fr}}}^2. \]

The damping forces of MR dampers can be presented as

\[ u = \begin{bmatrix} c_{\text{a}}(z_{\text{a}} - z_{\text{fr}}) + F_{\text{a}} \\ c_{\text{fr}}(z_{\text{fr}} - z_{\text{a}}) + F_{\text{fr}} \\ c_{\text{rr}}(z_{\text{rr}} - z_{\text{fr}}) + F_{\text{rr}} \end{bmatrix} = u(\Delta, I) \quad (3) \]

where \( \Delta \) represents the four piston velocities of the four MR dampers, while \( I \) denotes the electromagnet coil current.

On considering the time delay uncertainty of the MR suspension system, the control force is \( u = u(t - t_d) \), where \( t_d \) is the uncertainty time delay. Then equation (2) can be written:

\[ \dot{x} = Ax + Bu(t - t_d) + Lw. \quad (4) \]

### 2.2. Control model of the MR damper

In this study, a cylindrical type of MR damper is schematically shown in figure 2(a) and it has similar dimensions to the typical original damper of a passenger. The diameters of the outer and inner cylinders are 50 mm and 48 mm, respectively. The compressed length of the MR damper is 330 mm. When it is extended, it reaches 508 mm. Figure 2(b) shows the photograph of the four MR dampers manufactured for this work.

The MR damper operates in a mixed-mode of a valve mode and a direct-shear mode. The inner cylinder of the MR damper is divided into left and right chambers by the piston, and fully filled with the MR fluid. When the piston moves in the inner cylinder, the MR fluid flows through the gap between the inner cylinder and the piston. Application of a perpendicular magnetic field to the motion direction of the MR fluid causes a controllable shear viscosity of the MR fluid. As a result, the damping force of the MR damper can be adjusted by varying the strength of the magnetic field or the current in the coil. Obviously, the model of the relationship between the damping force and the piston velocity and the coil current is important in order to utilize effectively the MR damper, which can be described using equation (3).

Although the damping force of an MR damper can be easily computed by equation (3) for a determined current, it is difficult to calculate inversely the current when the damping force is known since the relation of the input current and
the damping force is highly nonlinear. Adequate modeling of the MR damper is essential for the accurate prediction of the behavior of the controlled system. As a result, a model is developed to accurately reproduce the behavior of the MR damper and an experiment is set up to obtain the dynamic data necessary to identify its model parameters.

Tests are conducted on the MR dampers. A hydraulic actuator is used to drive the dampers, and the displacement and forces are measured. The velocities are calculated using the central differences approximation. Different sinusoidal frequencies and amplitude command signals are used. In this study, the excitation frequencies are 0.6369, 1.9108 and 3.8197 Hz and the displacement amplitudes are 25 mm, respectively, which result that the peak velocity of the piston rod are 0.1, 0.3 and 0.6 m s$^{-1}$, respectively. The applied input current is from 0 to 2 A in increments of 0.4 A. Typical hysteresis loops of the MR damper obtained through experimental testing are provided in figure 3. Figures 3(a) and (b) show the relations between damping force and piston rod displacement and the relations between damping force and piston rod velocity at a peak velocity of 0.6 m s$^{-1}$ under five different input currents for the MR dampers of the front suspension.

The polynomial model is adopted in this study, which was first proposed by Choi et al [25], which provided a convenient and effective choice to calculate the desirable damping force in an open-loop control system. In this model, the hysteresis loop is divided into two regions: positive acceleration (lower loop) and negative acceleration (upper loop), which can be fitted by the polynomial with the power of the piston velocity. Thus the damping force of the MR damper can be written

$$f_{\text{MR}} = \sum_{i=0}^{9} a_i v^i$$

(5)

The efficiency $a_i$ can be linearly approximated with respect to the input current as follows:

$$a_i = b_i + c_i I$$

Therefore, the damping force can be expressed by

$$f_{\text{MR}} = \sum_{i=0}^{9} (b_i + c_i I) v^i$$

(6)

where the coefficients $b_i$ and $c_i$ are obtained from the fitting of the experimental data.

To verify the obtained polynomial model, the measurement and simulation under five operating conditions are compared in figure 3 with an excitation frequency 3.8197 Hz and amplitude ±25 mm. From this figure, it can be seen that the model of the MR damper can accurately predict the behavior of the MR damper. Once the piston velocity and the desired force determined by the control strategy are known, the control current can be calculated from equation (7):

$$I = \frac{f_{\text{MR}} - \sum_{i=0}^{9} b_i v^i}{\sum_{i=0}^{9} c_i v^i}$$

(7)

where $I$ is the MR damper input current and $f_{\text{MR}}$ is the desired damping force determined by the later designed control strategy.
Figure 3. Comparison of polynomial model and experimental results (3.8197 Hz, 25 mm): (a) force versus displacement; (b) force versus velocity.

Figure 4. HSIC structure for MR semi-active suspension system.

2.3. Development of HSIC based on SMIS

Zhou et al first proposed the original HSIC algorithm in 1983 [26]. The theory has been applied successfully in many perplexing processes and plants (such as delay systems, multivariable systems, nonlinear systems, etc). In recent years, the theory has made great progress by absorbing the new research results, and schema theory, of cognitive science. As a result, a new intelligent control algorithm, HSIC based on SMIS, has been developed [27–29].

In the authors’ previous work [30, 31], the original HSIC was proposed for the half-car model and the quarter-car model featuring an MR damper. In this study, the HSIC based on the SMIS is proposed for a full-car model with four MR dampers with time delay uncertainty. As the first step of control design, we can look at a running car as a moving robot. The moving robot may exhibit one or more motions among heave, pitch, and roll motions. For the sake of the disturbance of the road and the driver’s maneuvers, it is difficult to suppress these vibrations with a single control model. Naturally, a controller with a multimode control strategy may be effective in controlling the different attitudes of the car body. According to the motion characteristics of the car body, motion attitudes of a running car are eight in figure 4. The first motion attitude is an ideal attitude with little change of motion attitude. Under that condition good ride comfort and stability can be guaranteed and it is feasible to propose an open-loop control strategy. The attitudes from the second to the eighth motion attitude are the coupling of one or more motions of heave, pitch or roll motion due to the road input or driver’s maneuvers. It is effective to propose to combine skyhook control with proportional plus differential control schemes.

2.3.1. The entire structure of HSIC based on SMIS for the MR suspension system. The HSIC based on SMIS with two levels of hierarchical structure is shown in figure 4. In this figure, the MR semi-active suspension system is excited by the road. The HSIC consists of two levels, the running level and the parameter adjustment level. At the top level, the parameter adjustment level adaptively modifies the control parameters of the running level when the running road condition is
changed. At the running level, eight SMIS automatically switch according to the condition of the running car and execute real-time control. In this figure, the SS denotes the sensed schema, the MS is the motion schema and the AS denotes the association schema. From $S_{P, S}$ to $S_{M, S}$ and from $S_{M, S}$ to $S_{A, S}$ we denote the eight sensed schemas, eight motion schemas and eight association schemas with respect to eight attitudes, respectively. Both two levels combine a unit controller with the structure of a typical high-order production system. For each level in HSIC, the control problem solving is in reality a combination of a qualitative and quantitative double mapping information process and decision procedure. The procedure of designing the intelligent control algorithm is the process of setting up a characteristic model and multimode control model. It can be described by a production rule in the form of $IF$ (condition) $THEN$ (result) (condition is characteristics and result is decision). The two level designs will be discussed in turn.

2.3.2. Running control level. At this level, eight sensor motor intelligent schemas (SMIS) are designed to control directly the corresponding attitude according to the characteristic mode. Each SMIS consists of sensed schema, motion schema and association schema. The design course of the eighth motion attitude including heave, pitch and roll motions is similar to others. Therefore, the design of the eighth motion attitude is only discussed in the following.

2.3.2.1 Sensed schema. To determine the motion attitude of the running car, a characteristic set can be selected:

$$S_{P, S} = (R_S, Q_S, K_S, \otimes, \Phi)$$

where $R_S \in \sum_{14}^8$ is the input information set, $Q_S \in \sum_9^8$ is the characteristic primitive set and $K_S \in \sum_8^{8 \times 9}$ is the relation matrix between the input information set and the characteristic primitive set and $\Phi_S \in \sum_8^8$ is the characteristic model set. $\otimes$ is the operational symbol.

The characteristic primitive set is selected to effectively show the attitudes of the running car and can be expressed as $Q_S = \{ q_{s1} \mid \text{condition} \}$, where $q_{s1} \in \sum_8^8$, $\otimes$ is the characteristic attitude.

The relation matrix $K_S$ can be determined defleccting from or approaching the target attitude:

$$K_S = \begin{pmatrix}
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1
\end{pmatrix}$$

As a result, the sensed characteristic model can be obtained:

$$\Phi_S = K_S \otimes Q_S = \begin{pmatrix}
\phi_{s1}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi > 0, \varphi < 0, \theta > 0, \\
\phi_{s2}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi < 0, \theta < 0, \\
\phi_{s3}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi > 0, \theta < 0, \\
\phi_{s4}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi < 0, \theta > 0, \\
\phi_{s5}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi < 0, \theta < 0, \\
\phi_{s6}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi < 0, \theta > 0, \\
\phi_{s7}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi > 0, \theta > 0, \\
\phi_{s8}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi > 0, \theta > 0, \\
\phi_{s9}(\mid \text{condition} \mid) > \delta , [ \theta ] > \delta , [ \varphi ] > \delta , [ z_1 ] > 0, \varphi > 0, \theta > 0
\end{pmatrix}$$

(10)

2.3.2.2 Motion schema. The eighth motion schema can be presented as

$$S_{M, S} = (R_S, P_S, L_S, \Psi_S, U_S)$$

(11)

where $R_S \in \sum_{14}^8$ is the input information set, $P_S \in \sum_8^8$ is the control mode primitive set, $L_S \in \sum_8^{8 \times 8}$ is the relation matrix, $\Psi_S \in \sum_8^8$ is the control mode set and $U_S$ is the control output.

The eighth motion attitude involves heave, pitch and roll motion. The control inputs are adjustable damping forces of four MR dampers. Therefore, it is reasonable to calculate independently the damping force required by the heave, pitch and roll motion, respectively.

According to the sensed schema, the heave motion can be classified into two situations. One is when the car body approaches the target place and a skyhook control strategy is proposed:

$$F_{dH} + F_{dHR} + F_{dHr} + F_{dHR} = -k_{sky}C_{sky-z}z$$

(12)

where $k_{sky}$ denotes an adjustable factor and $C_{sky-z}$ is the skyhook damping coefficient to reduce the heave vibration of the vehicle body.

The other is when the car body departs from the target place, when a proportional plus differential control scheme is proposed:

$$F_{dH} + F_{dHR} + F_{dHr} = -k_{pd}(K_{p-z}z + K_{d-z}z)$$

(13)

where $k_{pd}$ denotes an adjustable factor, and $K_{p-z}$ and $K_{d-z}$ are the proportional and derivative gains of the proportional plus derivative control to depress the heave vibration of the vehicle body.

To restrain pitch motion, we can get similar equations. When the car body approaches the target place

$$F_{dH}a + F_{dHR}b + F_{dHr}b = -k_{sky}C_{sky-p}p$$

(14)

where $C_{sky-p}$ is the skyhook damping coefficient to reduce the pitch vibration of the vehicle body.

When the car body departs from the target place

$$F_{dH}a - F_{dHR}a + F_{dHr}b + F_{dHR}b = -k_{pd}(K_{p-p}p + K_{d-p}p)$$

(15)
where $K_{p-q}$ and $K_{d-v}$ are the proportional and derivative gains of the proportional plus derivative control to reduce the pitch vibration of the vehicle body.

By the same principle, the damping forces to restrain roll motion can also be calculated as follows:

$$ F_{d_j} \frac{w}{2} = F_{d_j} \frac{w}{2} - F_{d_j} \frac{w}{2} - F_{d_j} \frac{w}{2} = -k_{pd}(K_{p-q} \theta + K_{d-q} \theta) \tag{16} $$

$$ F_{d_j} \frac{w}{2} = F_{d_j} \frac{w}{2} + F_{d_j} \frac{w}{2} - F_{d_j} \frac{w}{2} = -k_{pd}(K_{p-q} \theta + K_{d-q} \theta) \tag{17} $$

where $C_{sky-\theta}$ is the skyhook damping coefficient, and $K_{p-q}$ and $K_{d-q}$ are the proportional and derivative gains of the proportional plus derivative control to reduce the roll vibration of the vehicle body.

We are unable to calculate the four damping forces from three equations. On neglecting torsion of the car body, an equation can be supplemented:

$$ F_{d_j} \frac{w}{2} - F_{d_j} \frac{w}{2} - F_{d_j} \frac{w}{2} = 0. \tag{18} $$

According to the sensed schema, an equation group consists of (12), (14), (16) and (18) or (13), (15), (17) and (18). The equation group is resolved to get a control force vector $p_1 = (F_{d_j}, F_{d_j}, F_{d_j}, F_{d_j})^T$.

2.3.2.3 Association schema. The association schema can be described as

$$ \Omega_8 : \Phi_8 \rightarrow \Psi_8, \quad \Omega_8 = \{\omega_{11}, \omega_{12}, \ldots, \omega_{18}\} \tag{19} $$

where $\omega_{ij}$: IF $\Phi_{8j}$ THEN $\Psi_{8i}$ ($j = 1, \ldots, 8$).

Therefore, the whole schema of the eighth attitude can be written as

$$ S_{KG,8} = \{S_{P,8}, S_{M,8}, S_{A,8}\}. \tag{20} $$

2.3.2.4 Formulation of all schemas. After the other seven schemas of the running level are formulated, all the schemas including all sensed schemas, all motion schemas and all associated schemas can be described together.

All sensed schemas are

$$ S_p = (R, \quad Q, \quad K, \quad \otimes, \quad \Phi) \tag{21} $$

in which $R \in \sum^{14}, \quad Q \in \sum^{6}, \quad K \in \sum^{8 \times 6}, \quad \Phi \in \sum^8$.

The characteristic primitive set is

$$ Q = \{q_1 \mid z > \delta_z, \quad q_2 \mid \theta > \delta_\theta, \quad q_3 \mid \psi > \delta_\psi, \quad q_4 \mid z \leq \delta_z, \quad \theta \leq \delta_\theta, \quad \psi \leq \delta_\psi\} \tag{22} $$

and the relation matrix is

$$ K = \begin{pmatrix}
0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix} $$

As a result, the characteristic mode set can be determined:

$$ \Phi = K \otimes Q = \{\phi_1 \mid z \leq \delta_z, \quad \theta \leq \delta_\theta, \quad \psi \leq \delta_\psi\} \tag{23} $$

2.4. Time delay uncertainty compensation

In previous study, a Smith’s predictor was proposed to compensate for the certain time delay of the MR damper. But the actual time delay of the MR damper is an uncertain variable. To compensate for the time delay uncertainty of MR dampers, a backpropagation network with four layers and nodes $N_1 - N_2 - N_3 - N_4$ (In this study, $N_1 = 4, N_2 = 8,$

$$ S_M = (R, \quad P, \quad L, \quad \Psi, \quad U) \tag{24} $$

where $P \in \sum^8, \quad L \in \sum^{8 \times 8}, \quad \Psi \in \sum^8$.

The control mode primitive set is

$$ P = \{S_{KG,1}, S_{KG,2}, S_{KG,3}, S_{KG,4}, S_{KG,5}, S_{KG,6}, S_{KG,7}, S_{KG,8}\}. \tag{25} $$

If the mode computation matrix $L = I_{8 \times 8}$ is an identity matrix with eighth order, the control mode set can be obtained:

$$ \Psi = LP^T. \tag{26} $$

The all associated schema is

$$ S_A = \{\Omega : \Phi \rightarrow \Psi\}, \quad \Omega = \{\Omega_1, \Omega_2, \Omega_3, \Theta_1, \Theta_2, \Theta_3, \Theta_4\} \tag{27} $$

$$ \Theta_j : IF \Phi_j, \then \Psi_j (j = 1, 2, \ldots, 8). $$

Consequently, all SMIS of the running level for the MR suspension system can be presented:

$$ S_{KG} = (S_P, S_M, S_A). \tag{28} $$

2.3.3. Parameter adjustment level. To increase the adaptive ability of HSIC, two adjustable factors $k_{sky}$ and $k_{pd}$ are introduced in the running control level, which are used in equations (12)–(17). The adjusting rule can be written:

If $RMS(a) < RMS(a_0)$

$$ k_{sky} = \frac{RMS(a_0)}{RMS(a_0)}. \tag{29} $$

Else

$$ k_{pd} = \frac{RMS(a_0)}{RMS(a_0)}. \tag{30} $$

where $RMS(a_0)$ and $RMS(a)$ are the root-mean-square vertical acceleration of the car body during a period of time and $a_0$ is the reference vertical acceleration of the car.
$N_3 = 8, N_4 = 1$) is adopted. Correspondingly, the control outputs of the HSIC based on SMIS are adaptively adjusted during each control period. The control frequency is determined by the response time of the MR damper, the most concerned frequency range of vehicle vibration and the cost of hardware. In our previous work, the response time of the MR damper is about 25 ms and the most concerned frequency range of vehicle vibration is below 30 Hz. As a result, a 200 Hz controller frequency is selected in this study.

The mapping relationship of the neural network model is described as

$$y_M(t + 1) = f_M(u(t - t_d), u(t - t_d - 1), \ldots, u(t - t_d - m), y(t), \ldots, y(t - n))$$

where $m$ and $n$ denote the orders of the MR suspension system and sampling numbers respectively, and defining

$$s^T = (s_1, \ldots, s_N)^T = (u(t - t_d), u(t - t_d - 1), \ldots, u(t - t_d - m), y(t), \ldots, y(t - n))^T.$$  

The activation functions in the last three layers of the neural network model are, respectively

$$f_{i_j} = 1 \left/ \left\{1 + \exp \left[ - \left( \sum_{j=1}^{N_1} \theta_{i_j} x_i + q_{i_j} \right) \right] \right\} \right.$$  

$$f_{2k} = 1 \left/ \left\{1 + \exp \left[ - \left( \sum_{j=1}^{N_2} \theta_{2jk} f_{i_j} + q_{2k} \right) \right] \right\} \right.$$  

$$y_M(t + 1) = 1 \left/ \left\{1 + \exp \left[ - \left( \sum_{k=1}^{N_3} \theta_{3k} f_{2k} + q_{3} \right) \right] \right\} \right.$$  

The output of the conventional control system with the uncertainty time delay $t_d$ is

$$y(t + 1) = L^{-1}[G(s)e^{-t_d s}] = f(u(t - t_d), u(t - t_d - 1), \ldots, u(t - t_d - m), y(t), \ldots, y(t - n)).$$

The output without time delay is

$$y_r(t + 1) = L^{-1}[G(s)] = f_r(u(t), \ldots, u(t - m), y_r(t), \ldots, y_r(t - n))$$

where $L^{-1}[\bullet]$ is the inverse Laplace transform and $G(s)$ represents the transfer function of the system in equation (2).

The predicted value of the output without time delay is given by the neural network system without the time delay is given by the neural network model and used to realize a compensating control. The network is trained by the sequence of input–output samples. Using the same output of the HSIC based on SMIS, we can obtain the predicted value of the output without time delay as follows:

$$y_r(t + 1) = f_M(u(t), u(t - 1), \ldots, u(t - m), y_r(t), \ldots, y_r(t - n)).$$

The compensating error is

$$\tilde{e} = y_r(t + 1) - y(t + 1).$$

To determine the weights of the neural network, the time delay samples in some working conditions are experimentally tested in [32]. The offline learning results of the compensator can be used as a reference model of the MR suspension system. The weights are modified by the index (38) using the principle of error gradient descent.

3. Numerical simulation

In this section, the performance of the semi-active suspension system is evaluated numerically using the proposed HSIC based on SMIS. The nominal model parameters for full-car model are shown in table 1.

In order to evaluate the proposed compensation control strategy, numerical simulation of the full-car suspension system for various roads and vehicle speeds are carried out in MATLAB and SIMULINK using a fourth-order Runge–Kutta integration scheme. The numerical conditions consist of a bumpy road at a speed of 30 km $^{-1}$ and a B class road at a speed of 90 km $^{-1}$.
Figure 7. The time history of vertical acceleration of the car body under bumpy input at a vehicle speed of 30 km h\(^{-1}\).

### Table 1. Simulation parameters of a type of saloon car.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sprung mass, ( m_s )</td>
<td>745.2 kg</td>
</tr>
<tr>
<td>Roll axis moment of inertia, ( I_{xx} )</td>
<td>375.2 kg m(^2)</td>
</tr>
<tr>
<td>Pitch axis moment of inertia, ( I_{yy} )</td>
<td>768.8 kg m(^2)</td>
</tr>
<tr>
<td>Front unsprung mass, ( m_{ufl}, m_{ufrr} )</td>
<td>25.35 kg m(^2)</td>
</tr>
<tr>
<td>Rear unsprung mass, ( m_{url}, m_{urr} )</td>
<td>68.8 kg m(^2)</td>
</tr>
<tr>
<td>Front suspension spring stiffness, ( k_{fl}, k_{fr} )</td>
<td>30,000 N m(^{-1})</td>
</tr>
<tr>
<td>Rear suspension spring stiffness, ( k_{rl}, k_{rr} )</td>
<td>32,500 N m(^{-1})</td>
</tr>
<tr>
<td>Length of wheelbase, ( d )</td>
<td>1.277 m</td>
</tr>
<tr>
<td>Length between front wheel and C.G. of vehicle, ( a )</td>
<td>1.1161 m</td>
</tr>
<tr>
<td>Length between rear wheel and C.G. of vehicle, ( b )</td>
<td>1.2319 m</td>
</tr>
<tr>
<td>Tire spring stiffness, ( k_{ufl}, k_{ufrr}, k_{url}, k_{urr} )</td>
<td>181,000 N m(^{-1})</td>
</tr>
<tr>
<td>Passive damping coefficient of OEM damper, ( c_{fl}, c_{fr} )</td>
<td>1039.7 N m(^{-1}) s(^{-1})</td>
</tr>
<tr>
<td>Passive damping coefficient of OEM damper, ( c_{rl}, c_{rr} )</td>
<td>2638.0 N m(^{-1}) s(^{-1})</td>
</tr>
</tbody>
</table>

The dimensions of the bumpy road are determined by measuring the actual speed bump in figure 14 and shown in figure 5. The random road inputs of the two front wheels have the road power spectral density [34]

\[
G_x(n) = G_x(n_0) \left( \frac{n}{n_0} \right)^{-w}.
\]  

(39)

If the vehicle velocity is \( v_0 \), then the delay time of the rear wheel excitations compared to the front wheel excitations is \( \Delta t = l/v_0 = (a + b)/v_0 \). Thus, the time serials of the four-wheels road model for a B class road at a speed of 60 km h\(^{-1}\) via simulation are shown in figure 6 according to equation (39).

For comparison, the numerical simulations of the semi-active suspension system with the HSIC without time delay compensation are carried out also under the same situations of that for the HSIC with time delay compensation. The time histories of vehicle body vertical acceleration and pitch angular acceleration under bump input at a vehicle speed of 60 km h\(^{-1}\).
30 km h\(^{-1}\) are shown in figures 7 and 8. Correspondingly, the control force tracking results of two control strategies are given in figure 9 and the force tracking errors are shown in figure 10. The power spectral density (PSD) of heave vibration acceleration of the vehicle for a random road with a speed of 60 km h\(^{-1}\) is shown in figure 11.

It is well known that the first peak-to-peak acceleration value has great influence on a human’s ride comfort when a car is running across a speed bump. It can be seen that the MR suspension with two designed controllers can reduce the peak-to-peak vertical or angular acceleration compared with the passive one. The MR suspension system with the HSIC based on SMIS has better control performance than that with the HSIC without considering the time delay uncertainty of the MR damper. The reason is that the tracking force errors of the MR damper with the HSIC based on SMIS are smaller than those without considering the time delay uncertainty of the MR damper.

In addition, the response of the sprung mass acceleration for a random road is shown in figure 11. For brevity, only the response of the acceleration PSD at a speed of 60 km h\(^{-1}\) on a B class road is presented. It can also be seen that the semi-active suspension systems via the two control strategies have good performance in sprung mass acceleration compared to that of the passive suspension system. Considering the human sensitivity frequency range of 4–8 Hz, the semi-active suspension system via the HSIC based on SMIS can achieve a better improvement in ride comfort compared to that via the HSIC without considering the time delay uncertainty.

4. Field test results and discussion

To experimentally validate the ride comfort quality benefits of the HSIC based on SMIS considering the time delay uncertainty of the MR damper, a vehicle was equipped with four MR dampers for a field test. The vehicle for the field test is a 1342 cc middle-sized passenger car (figure 12). It has a total length of 4215 mm, total width 1675 mm, total height 1375 mm, wheelbase 2610 mm, front track and rear track of 1470 mm and a weight of 1020 kg. Four MR dampers are easy to install in place of the stock dampers due to their dimensional similarities.

For this research, eight accelerometers are used to measure the vibration of the sprung mass and unsprung mass. Four
accelerometers of the eight are used to measure the vertical vibration acceleration of the unsprung mass and the other four are used to sense the vertical vibration acceleration of the four corners of the vehicle body. From this information, relative suspension velocity, displacement and velocity of the vehicle body can be calculated. Vehicle ride quality is assessed using the eight accelerometers and a three-axes B&K comfort pad. When measured data from the sensors are sent to the AutoBox through an A/D channel, vehicle movements are analyzed based on the measured data every 1 ms by the AutoBox. Desired damping forces for improving ride quality are calculated by the AutoBox. Control input is determined by the HSIC algorithm implemented in the AutoBox downloaded from MATLAB 6.5/SIMULINK from the computer, and the input signal is sent to the current driver through the D/A channel of the AutoBox, and then the amplified input current is applied to the MR damper. During tests, various sensor information is transferred to the data acquisition system for evaluation via a chovr box. For implementation of the HSIC algorithm, vertical velocity and displacement of the body are numerical integrated from the measured data by an accelerometer located in the suspension.

The accelerometer signals have DC offsets; the removal of the DC component is critical because the acceleration signals are integrated to obtain the absolute velocity or displacement. If a signal with a DC offset is integrated, the value of the integral goes to infinity as \( t \to \infty \). To combat this problem, the signal is passed through a transfer function of the form [33]

\[
H(s) = \frac{s}{(s + 0.2 \times 2 \times \pi)^2}. \tag{40}
\]

Figure 13 shows the Bode plot for this transfer function. From this figure, it can be seen that the transfer function looks like a high-pass filter at low frequencies. The transfer function looks like an integrator at frequencies above 0.2 Hz.

Testing was conducted for a variety of terrains and operating conditions. The results reported here are taken on a bumpy road and a random road. Both the vehicle speed on the bumpy road and the random road are maintained at 30 km h\(^{-1}\) and 60 km h\(^{-1}\), respectively. The random road contains many asphalt patches, bridge abutments, concrete expansion joints and raised manhole covers. Multiple trials were conducted under these conditions to ensure repeatability of test results.

The test results under bump excitation are presented in figures 15 and 16 by using the front left vertical acceleration of
the vehicle body and the pitch angle acceleration of the vehicle body at the C.G. calculated by the four accelerations of the four corners of the vehicle body. By adopting the MR suspension with two HSIC strategies, the vibration of the vehicle is rapidly reduced after passing the bump. It is clearly observed that the first peak-to-peak acceleration of heave or pitch motion of the vehicle body, which often induces strong discomfort in the human body, is reduced up to 35% over a passive suspension system, as shown in the two figures. In addition, the MR suspension via the HSIC based on SMIS considering the time delay uncertainty of the MR damper has smaller peak-to-peak acceleration values than that via the original HSIC considering the fixed time delay of the MR damper.

From the test results of a random road in figure 17, it can be seen that vibration energy of the vehicle body was considerably reduced, up to 60% at the first resonance frequency area via MR suspension with two HSIC control methods. The ride quality has been improved in the sensitive frequency range (4–8 Hz) of the human body. In the high frequency range over the second resonance frequency, the MR suspension cannot achieve any improvement of ride quality and even worsens the vibration of the vehicle. Fortunately, the effect on the human body in this frequency range is not as intense as that in the frequency range between 4 and 8 Hz for vertical vibrations of the car body [35]. Moreover, the MR suspension with the HSIC based on SMIS has a better ride quality improvement than that with the original HSIC. Table 2 shows the comparison of root-mean-square (RMS) acceleration collected from the above nine sensors. The nine sensors represent nine sites on the vehicle including left up (FL up), front right up (FR up), rear left up (RL up), rear right up (RR up), front left down (FL down), front right down (FR down), rear left down (RL down), rear right down (RR down) and driver’s seat. According to table 2, it can be found that the RMS acceleration of the sprung mass has been greatly reduced by over 24% and RMS acceleration of the unsprung mass has been slightly decreased. Furthermore, it can also be seen that the performance improvement of the MR suspension system made under bumpy road input is superior to that made under random input. This can be explained because the dynamic behavior of the MR suspension caused by the time delay of the MR damper under the determined road disturbance is different from that under the random road disturbance, and the time delay has a bigger influence on the dynamic behavior of the MR suspension on the determined road.

Comparing the numerical results (figures 7, 8, 11) and the results of road tests (figures 15–17), it can be seen that the values of vibration acceleration from the simulation are different from those from the road tests. This may be due to modeling errors in some factors such as frictional forces of spring stiffness which are not considered in the simulation. However, similar conclusions can be drawn from both the simulation and the road tests.

<table>
<thead>
<tr>
<th>Index</th>
<th>FLup (m s$^{-2}$)</th>
<th>Frup (m s$^{-2}$)</th>
<th>RLup (m s$^{-2}$)</th>
<th>RRup (m s$^{-2}$)</th>
<th>FLdown (m s$^{-2}$)</th>
<th>Frdown (m s$^{-2}$)</th>
<th>RLdown (m s$^{-2}$)</th>
<th>RRdown (m s$^{-2}$)</th>
<th>Comfort (m s$^{-2}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passive</td>
<td>0.8901</td>
<td>0.9729</td>
<td>1.037</td>
<td>1.0361</td>
<td>4.5802</td>
<td>4.3943</td>
<td>8.3059</td>
<td>8.5059</td>
<td>0.7218</td>
</tr>
<tr>
<td>HSIC based on SMIS</td>
<td>0.6535</td>
<td>0.732</td>
<td>0.7867</td>
<td>0.7668</td>
<td>3.9802</td>
<td>3.8112</td>
<td>7.374</td>
<td>7.4792</td>
<td>0.5215</td>
</tr>
<tr>
<td>Effect</td>
<td>26.58%</td>
<td>24.76%</td>
<td>24.14%</td>
<td>25.99%</td>
<td>13.10%</td>
<td>13.27%</td>
<td>11.22%</td>
<td>12.07%</td>
<td>27.75%</td>
</tr>
<tr>
<td>HSIC</td>
<td>0.6755</td>
<td>0.7396</td>
<td>0.7968</td>
<td>0.7931</td>
<td>4.0672</td>
<td>3.8534</td>
<td>7.1888</td>
<td>7.3806</td>
<td>0.5361</td>
</tr>
<tr>
<td>Effect</td>
<td>24.11%</td>
<td>23.98%</td>
<td>23.16%</td>
<td>23.45%</td>
<td>11.20%</td>
<td>12.31%</td>
<td>13.45%</td>
<td>13.23%</td>
<td>25.73%</td>
</tr>
</tbody>
</table>

Figure 17. PSD of comfort pad acceleration at 30 km h$^{-1}$.

5. Conclusion

After modeling the full-car dynamical model featuring four MR dampers considering the time delay uncertainty, an HSIC based on SMIS is proposed. With an idea of a running car with eight motion attitudes, each SMIC is formulated in order. A neural network model is proposed to compensate for the time delay uncertainty of MR dampers using the experimental data. Simulations and field tests are performed under various road conditions. It has been demonstrated through bumpy and random road tests that ride comfort of the vehicle can be much improved by controlling the semi-active MR suspension system via two HSIC strategies. The improvement in stability is also demonstrated by reducing the pitch motion and unsprung mass motion. In addition, the MR suspension system with the HSIC based on SMIS can avoid the effect of time delay uncertainty and have a better ride quality than that with the original HSIC. In this study, only the time delay caused by the electrical part and the mechanical part is discussed: the time delay caused by the control electronics will be studied in the near future in order to investigate the effect
of the entire time delay of the MR suspension system on the control performance.

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